

Housing Supply and Affordability*

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We examine how housing supply constraints affect housing affordability, which we define as the quality-adjusted price of housing services. Our dynamic model predicts that supply constraints will increase the price of housing services by only about half as much as the purchase price of a home, since the purchase price responds to expected future increases in rent as well as contemporaneous rent increases. In the model, households respond to changes in the price of housing services by altering their consumption and location choices, further reducing the implications of supply constraints for housing expenditures. Next, we estimate the effects of common measures of supply constraints on housing outcomes using data from US metropolitan areas from 1980 to 2016. We find sizeable effects of supply constraints on house prices, but modest-to-negligible effects on rent, unit size, lot size, location choice within metropolitan areas, sorting across metropolitan areas, and housing expenditures. We conclude that housing supply constraints distort housing consumption and affordability much less than their estimated effects on house prices would suggest.

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1. Introduction

A large and growing literature has documented a strong connection between housing supply constraints and house prices (Glaeser and Gyourko (2003), Quigley and Raphael (2005), Ihlanfeldt (2007), Zabel and Dalton (2011), Hilber and Vermeulen (2016), Albouy and Ehrlich (2018)). Much less work has analyzed how these effects map into changes in housing affordability.¹ One likely reason for this gap is that housing affordability is defined in many different ways in the academic and policy realms. We define housing affordability as the quality-adjusted price of housing services. This measure is more relevant than the purchase price of a home for understanding how supply constraints affect household welfare. Through their effects on the price of housing services, supply constraints can change welfare by altering household consumption and location decisions. Thus, to obtain a comprehensive view of the effects of housing supply constraints on housing affordability and households, this paper examines the effects of these constraints on house prices, rents, housing consumption, and household location.

Our analysis begins with a dynamic model in which households choose a level of housing services and whether to live in an unregulated city or in a city with supply constraints that explicitly limit how fast it can grow. Developers combine structure and lots to supply housing services given local constraints and household demand. Supply constraints raise the purchase price of homes by more than rent (the spot price of housing services) because supply constraints increase expected growth in future rent as well as the current level of rent.² In response to the higher price of housing services, households with a given income choose to live on smaller lots, and fewer households choose the constrained city. Other housing outcomes depend on whether the constraint limits the city growth by land area or by population, and also on parameters of the housing production function and the household utility function. In a calibration exercise, we find that the effects on rent are about half of the effects on the purchase price of housing. Meanwhile, the effects on housing expenditures are even smaller than the effects on rent because households make a variety of adjustments—most of which are qualitatively small—to their consumption and location decisions.

¹ A few studies have found some correlation between regulation and affordability as measured by rent relative to median metropolitan area income (Somerville and Mayer 2003, Pendall 2000). Glaeser and Gyourko (2003) argue that housing affordability should be assessed by the level of house prices relative to construction costs, and show that metropolitan areas with longer permitting times more regulated metropolitan areas have a larger fraction of homes with prices substantially greater than construction costs. Albouy and Ehrlich (2018) estimate the effect of regulations on metropolitan amenities and construction productivity and find that the total effect of regulations on social welfare is negative.

² Gyourko, Mayer and Sinai (2013) also develop a model in which an inelastic supply of housing raises house prices more than rent, although they do not derive the effects of supply constraints on rent. While their model of consumer choice is static, ours is dynamic, giving us a richer framework to estimate the quantitative effects on rents relative to prices.

Next we evaluate the model's predictions using variation across metropolitan areas in two measures of housing supply constraints that are standard in the literature. As a measure of land availability, we use geographic constraints calculated by Saiz (2010), which are derived from the fraction of land on a steep slope or covered by water. As a measure of regulations that restrict the growth of the housing stock, we use the Wharton Residential Land Use Regulation Index, which is composed of a range of types of regulations from a survey of local government officials that was conducted in 2006 (Gyourko, Saiz and Summers 2008).

Importantly, regulations do not arise randomly across areas, and the regulatory environment is likely correlated with characteristics of an area that affect the housing market outcomes in which we are interested (Davidoff 2016). Geographic constraints also might be correlated with omitted variables that affect housing outcomes. We address this endogeneity problem in three ways. The first is to focus on changes in housing market outcomes from 1980 to 2016, under the assumption that metropolitan areas with stricter regulations in the early 2000s experienced larger increases in the severity of regulation during this period. Section 3 provides evidence for this assumption. We also assume that geographic constraints became more binding over this period, which is consistent with the increasing density in metropolitan areas. Our second approach to mitigate endogeneity is to control for time-varying factors that might also be correlated with supply constraints and housing outcomes. Our third approach is to drop metropolitan areas that experienced persistently low demand over our sample period, as these locations are likely different from growing metros along many unmeasurable dimensions.

We begin our empirical analysis by estimating the effects of supply constraints on house prices and rent using property-level data from the 1980 Census and the 2012-2016 American Community Survey (ACS). Consistent with the model's predictions, the effects of the supply constraints on rent are about half the estimated effects on house prices. Moreover, the estimated effects on rent are small in absolute magnitude. For example, a metro with 2 standard deviation tighter regulation experienced 7 percentage point stronger rent growth over this 35-year period, which works out to less than $\frac{1}{4}$ percentage point faster growth per year. A few other studies have noted that rents tend to be less correlated with housing supply regulations than house prices (Malpezzi 1996, Malpezzi and Green 1999, Green 1999, Xing, Hartzell and Godschalk 2006), but they do not explain why this occurs or link the results to implications for housing affordability. Gyourko, Mayer and Sinai (2013) show that metropolitan areas with a tight housing supply and strong demand have higher ratios of prices to rent, but they do not look explicitly at the role of specific supply constraints, nor do they examine the effects on rent directly.

Next we examine the effects of supply constraints on a variety of housing consumption decisions: unit size, lot size, structure type, and household size. The first two outcomes are obtained from property tax records, and the last two outcomes are obtained from the Census and ACS data. We find small effects of supply constraints on all of these outcomes, and the standard errors are small enough that we can reject large negative effects.

Turning to effects on household location choices, in the Census and ACS data we find that regulatory constraints lead to slightly lower growth in the housing stock and a small amount of sorting by income and education. These results explain very little of the aggregate amount of sorting by income and education across metros that has occurred between 1980 and 2016. Geographic constraints reduce the number of housing units but do not appear to cause any sorting by income across areas. The amount of sorting by income that we find in our analysis is materially less than the amount found by Gyourko, Mayer and Sinai (2013), likely because they examine sorting into areas that have both a constricted supply and strong demand, whereas we focus solely on supply constraints.

We also estimate the effects of housing supply constraints on housing expenditures. These expenditures combine effects on housing costs with consumption and location decisions. We find that both constraints raise housing expenditures by a bit more than implied by the model, although still by less than the estimated effects on house prices.

Broadly speaking, our empirical results accord with the predictions of the model, in that we find much smaller effects of regulation on rent than on house prices and can reject large household adjustments along most dimensions. One interesting exception is the effects of geographic constraints, where the model predicts that most households remain in the city and occupy houses on much smaller lots, while spending no more on housing. In contrast, in the data, we find that lot sizes do not shrink in response to geographic constraints but that many households migrate out of the city. Moreover, the data suggest that household expenditures rise somewhat in more geographically constrained areas. These results are consistent with the possibility that minimum lot sizes and other regulatory constraints prevent households from adjusting their housing consumption as much as they would prefer, pointing to a potentially important interaction between geographic and regulatory constraints. Banzhaf and Mangum (2019) also find evidence that structural and regulatory constraints create frictions in housing consumption.

One issue that our model does not address is location choice within a metro area. We might observe little adjustment along the dimensions of housing consumption or metro-level sorting because households instead offset higher housing costs by choosing to live in neighborhoods

with relatively low land prices, such as those with long commutes. We look for evidence of this possibility by examining housing construction by Census tract from 1980 to 2016. We measure neighborhood amenities with distance to the metro central business district, average commute time, school quality and crime. While we find some evidence that geographic constraints (but not regulatory constraints) have increased the housing stock in areas with lower school quality and higher crime rates, they also appear to have led to a shift in construction to locations closer to the CBD with shorter commute times. All of these interactions are relatively small and, on net, it seems unlikely that household location choice within a metro is sufficient to obscure or offset large effects of supply constraints on the price of housing services.

In summary, we find that the effects of supply constraints on the price and quantity of housing services are substantially smaller than their effects on house prices. Because it is housing services, and not homeownership, that matters for welfare, our results suggest that the housing consumption and affordability distortions from supply constraints are much smaller than the effect on prices would suggest.

2. Model

2.1. Environment and equilibrium

There are two cities, R (for “regulated”) and F (for “free”). Time runs continuously from $t = 0$. The economy consists of N_t households, each living in one of these two cities. The utility of household i is

$$U_i = \int_{t_i}^{\infty} e^{-rt} \log(a_{i,j,t} v(c_{i,t}, h_{i,t})) dt,$$

where t_i is the time the household is born, r is the discount rate, $a_{i,j}$ is its taste for city j , $c_{i,t}$ is non-housing consumption, and $h_{i,t}$ is housing consumption. Flow utility from non-housing and housing consumption is Cobb-Douglas: $v(c_{i,t}, h_{i,t}) = c_{i,t}^{1-\alpha} h_{i,t}^{\alpha}$, where $\alpha \in (0,1)$. The household receives income y_i that is constant over time. The distribution of income across households has a probability density function f .

Households differ in their city tastes:

$$a_{i,j} = a_j e^{\frac{\epsilon_{i,j}}{\beta}},$$

where $\epsilon_{i,R}$ and $\epsilon_{i,F}$ vary as independent standard extreme value distributions and $\beta > 0$.³ We assume that city tastes are independent from household income.

³ McFadden (1973) demonstrates the useful properties of extreme value distributions in the context of logit choice models.

Each household is part of a “dynasty,” a collection of households with identical income and city tastes. At a given time, the dynasties contain the same number of households, and the number of households grows at a rate g . Each dynasty chooses cities and consumption levels for its households to maximize the sum of their utility. The dynasty can borrow against the future income of its households at a constant rate r , yielding the budget constraint

$$\int_0^{\infty} e^{-rt} \sum_{i \in d} (c_{i,t} + p_{j_i,t}(h_{i,t})) dt \leq \int_0^{\infty} e^{-rt} \sum_{i \in d} y_i dt,$$

where $p_{j,t}(h)$ is the rental price of h units of housing in city j at time t , and the price of non-housing consumption equals one. Although an artificial modeling device, the dynasty represents bequests between generations and preserves symmetry in the model between households who arrive at different times.

Competitive developers supply housing in each city using two inputs: land, l , and tradeable capital, q . Epple, Gordon, and Sieg (2010), Ahlfeldt and McMillen (2014), and Combes, Duranton, and Gobillon (2016) find that a constant returns to scale, Cobb-Douglas function of these two inputs approximates the production process for housing very well. Thus, we assume the following production function in our model:

$$h(l, q) = l^{1-\gamma} q^{\gamma},$$

where $0 < \gamma < 1$. To abstract from issues of durability, we allow developers to supply housing services in frictionless spot markets. The marginal flow cost of structure is k^q and the marginal flow cost of land assembly is k^l . These costs are identical across cities and constant over time.

Regulators in city R unexpectedly impose one of two restrictions on developers for all $t > 0$:

- The total number of separate housing units cannot grow at a rate greater than g^n .
- The total land used for housing cannot grow at a rate greater than g^l .

These rules come at the end of time 0, after developers and dynasties have made their initial decisions. The first restriction limits the speed at which developers may supply new housing, so it corresponds to delays in the permitting process as well as regulations such as permit limits that restrict the amount of new construction. Because each household lives in a separate housing unit, this regulation restricts the growth rate of the city’s population. In contrast, the second restriction limits the geographic expansion of the city, so it corresponds to geographic constraints on housing supply. It could also reflect some regulatory restrictions, such as open space requirements. In city F , the number of housing units and area of land used are unconstrained.

Developers must obtain a permit to supply a housing unit at time t . The endogenous permit price is $x_{j,t}$, with $x_{F,t} = 0$ due to the absence of regulations in city F . Unpermitted land

available for development in city j trades among developers at an endogenous spot price $p_{j,t}^l$, which also equals zero in F . Developer cost minimization pins down the rental price of housing:

$$p_{j,t}(h) = x_{j,t} + \gamma^{-\gamma}(1-\gamma)^{\gamma-1}(p_{j,t}^l + k^l)^{1-\gamma}(k^q)^\gamma h.$$

The price to buy housing outright equals the expected net present value of future rents:

$$p_{j,t}^{own}(h) = E_t \int_t^\infty e^{-r(t'-t)} p_{j,t'}(h) dt'.$$

Equilibrium consists of prices $p_{j,t}^l$, $x_{j,t}$, and $p_{j,t}(h)$ such that dynasties maximize utility subject to their beliefs and budget constraints, developers maximize profits while obeying the regulations in R , and the housing market clears in each city. At $t = 0$, dynasties expect prices that hold in an equilibrium without any regulation, while they expect the prices that hold in the regulated equilibrium for $t > 0$. Appendix A.4 characterizes equilibrium at $t = 0$.

2.2. Equilibrium effects of population constraints

To isolate the effect of the population constraint, we set $g^n < g$ so that the constraint binds, while assuming that g^l is sufficiently large so that the price of land in R equals zero. Proposition 1 describes household city choices given the path of permit prices.

Proposition 1 (sorting). *If $a_{R,i} < a_{F,i}$, household i always lives in F . If $a_{R,i} \geq a_{F,i}$, household i lives in R only while $t \leq t_i^*$, which solves*

$$\log\left(\frac{a_{R,i}}{a_{F,i}}\right) = \frac{x_{R,t_i^*}}{y_i - \bar{x}(x_{R,t_i^*})}$$

where $\bar{x}(x) \equiv \int_{(t|x_{R,t} \leq x)} (r-g)e^{-(r-g)t} x_{R,t} dt$.

According to the proposition, households with a greater taste for R live there until the permit price becomes too high. This threshold price is larger when the relative taste for R is greater or the household's income is higher. Because the threshold rises in income, regulation skews the city R income distribution to the right, inducing outmigration of poorer households.

In equilibrium, the number of households choosing R must equal the maximal number that city R allows. To calculate the former, we compute the number of households at t whose relative taste for R exceeds the right side of the equation in Proposition 1 for $x_i^* = x_t$. The latter comes from growing the initial population (appearing in Appendix A.2) by g^n . Equating these gives

$$e^{-(g-g^n)t} = \int_{\bar{x}(x_{R,t})}^\infty \frac{a_F^\beta + a_R^\beta}{a_F^\beta \exp\left(\frac{\beta x_{R,t}}{y - \bar{x}(x_{R,t})}\right) + a_R^\beta} f(y) dy.$$

This equation pins down $x_{R,t}$. In particular, $x_{R,t}$ must strictly increase over time, reflecting the increasingly binding nature of the population constraint. Proposition 2 proves this result, but it is easy to see because the left side decreases in t while the right side decreases in $x_{R,t}$. The increasing nature of the permit price means that regulation increases prices more than rents:

Proposition 2 (prices versus rents). *The permit price, $x_{R,t}$, strictly increases in t . The effect of regulation on rents,*

$$\frac{p_{R,t}(h)}{p_{R,0}(h)} - 1 = \frac{x_{R,t}}{\gamma^{-\gamma}(1-\gamma)^{\gamma-1}(k^l)^{1-\gamma}(k^q)^\gamma h}$$

is therefore less than the effect of regulation on ownership prices,

$$\frac{p_{R,t}^{own}(h)}{p_{R,0}^{own}(h)} - 1 = \frac{\int_t^\infty r e^{-r(t'-t)} x_{R,t'} dt'}{\gamma^{-\gamma}(1-\gamma)^{\gamma-1}(k^l)^{1-\gamma}(k^q)^\gamma h}$$

for all positive t and h .

Each household living in R subtracts some constant amount from its flow income to pay the permit price. This deduction corresponds to \bar{x} in Proposition 1. The remaining income goes toward structure, lot, and non-housing consumption. Due to Cobb-Douglas preferences and production, the shares of remaining income going to these purposes are $\alpha\gamma$, $\alpha(1-\gamma)$, and $1-\alpha$, respectively. Proposition 3 formalizes this argument.

Proposition 3 (housing characteristics). *Structure and lot sizes for household i in R are*

$$\begin{aligned} q_i^* &= \alpha\gamma(k^q)^{-1}(y_i - \bar{x}(x_i^*)) \\ l_i^* &= \alpha(1-\gamma)(k^l)^{-1}(y_i - \bar{x}(x_i^*)). \end{aligned}$$

Both $E(q_i^ | y_i)$ and $E(l_i^* | y_i)$ strictly increase in y_i at each t .*

Proposition 3 establishes two offsetting effects of regulation on housing characteristics. Holding income constant, regulation unambiguously decreases structure and lot sizes by increasing $\bar{x}(x_i^*)$. This mechanism is an income effect: the permit price makes households poorer, leading them to consume less housing. Offsetting the income effect is a sorting effect. Holding the characteristics conditional on income constant, the sorting of poor households out of city R drives up average characteristics in R because these characteristics increase in income. The net effect of regulation on the average structure and lot size in city R is ambiguous.

2.3. Equilibrium effects of geographic constraints

To isolate the effect of geographic constraints, we set $g^l < g$ so that the constraint binds, while assuming that g^n is sufficiently large so that the permit price in R equals zero. Proposition 4 describes household city choices given the path of permit prices.

Proposition 4 (sorting). *Household i lives in R only if*

$$\log\left(\frac{a_{R,i}}{a_{F,i}}\right) \geq \alpha(1-\gamma) \log\left(1 + \frac{p_{R,t}^l}{k^l}\right)$$

and lives in F when this inequality does not hold.

As with population constraints, geographic constraints lead some households with a higher taste for R to live in F . But different from the population constraints, this outmigration is independent of household income because of Cobb-Douglas preferences and production. The housing characteristics for households in R clarify this point:

Proposition 5 (housing characteristics). *Structure and lot sizes for household i in R are*

$$\begin{aligned} q_i^* &= \alpha\gamma(k^q)^{-1}y_i \\ l_i^* &= \alpha(1-\gamma)(p_{R,t}^l + k^l)^{-1}y_i. \end{aligned}$$

By driving up the marginal cost of assembled land ($p_{R,t}^l + k^l$), geographic constraints lead to smaller lot sizes. The proportional decrease in lot size is the same for all income groups, and coincides with the term on the right side of the inequality in Proposition 4. This result holds because of Cobb-Douglas preferences and production. Also because of Cobb-Douglas preferences and production, structure size does not depend on geographic constraints.

To solve for the equilibrium price of land, we equate the total lot sizes of households choosing R with the maximal size that city R allows. The former comes from Propositions 4 and 5, while the latter comes from growing the initial city land size (appearing in Appendix A.4) by g^l . This equation yields a closed-form solution for the land price:

$$p_{R,t}^l = k^l \left(\left(1 + \frac{(e^{(g-g^l)t} - 1) \alpha_R^\beta}{\alpha_F^\beta} \right)^{\frac{1}{\beta\alpha(1-\gamma)}} - 1 \right),$$

which strictly increases over time. Using this formula, we prove our final proposition:

Proposition 6 (prices versus rents). *The effect of geographic constraints on rents,*

$$\frac{p_{R,t}(h)}{p_{R,0}(h)} = \left(1 + \frac{p_{R,t}^l}{k^l} \right)^{1-\gamma},$$

is less than the effect of regulation on ownership prices,

$$\frac{p_{R,t}^{\text{own}}(h)}{p_{R,0}^{\text{own}}(h)} = \int_t^\infty r e^{-r(t'-t)} \left(1 + \frac{p_{R,t'}^l}{k^l} \right)^{1-\gamma} dt',$$

for all positive t and h .

2.4. Calibration

We calibrate the model to quantify the effects of supply constraints on rents, housing expenditures, housing characteristics, and city incomes given the effect of constraints on ownership prices. To perform this exercise, we need values for the various model parameters. The appendix gives details on how we quantitatively solve the model given parameter values.

We use a discount rate of $r = 0.05$. We set the income distribution, f , to a lognormal with mean \$50,000 and log standard deviation 0.96, which is the mean of the standard deviations of positive log household income in the 1980 and 2016 U.S. Census data samples. We take $\alpha = 0.25$ from Davis and Ortalo-Magné (2011), who find that this share of income is spent on rent in many cities from 1980 to 2000. We set β , which governs the distributions of preferences for R and F , equal to three, a value that is within the range estimated by Diamond (2016) by computing the elasticity of cross-city migration with respect to changes in wages and rents. We set $\gamma = 2/3$, share of construction expenditure on structure that Albouy and Ehrlich (2018) estimate. The ratio a_R/a_F pins down the initial relative size of city R . We set it to 1 so that the cities have identical populations absent regulations in R . The economy growth rate, g , equals 0.01, reflecting average annual population growth in the U.S. between 1980 and 2016.

The final parameters are g^n and g^l , which describe the supply constraints. We choose these parameters so that each constraint raises the ownership price of a constant-quality house (at the median of the quality distribution at time zero) by 10% over 30 years. This magnitude is convenient because in our empirical estimates below, we find that a one standard deviation tighter constraint is associated with about 10 percent faster price growth over a roughly 30-year period. This methodology gives us values of $g^n = 0.0092$ and $g^l = 0.0093$.

Table 1 reports changes in outcomes given this assumed price increase. The case of population constraints appears in column (1), while the results under land area constraints are in column (2). Under both supply constraints, the rent of the initial median unit rises far less than prices—by only about half. In other words, about half of the effect on ownership prices comes from anticipation of future rent increases that the supply constraints will continue to cause. Figure 1 illustrates this result by showing the evolution of prices and rents in response to the population constraint. Initially, rent is unchanged because the population constraint only affects future growth. But prices jump by about 4 percent in response to anticipated future rent increases. Over time, prices and rents rise by similar amounts, so that the net increase in prices remains larger. Although the differential between prices and rents becomes a smaller fraction of rent as time goes on, it is still quite substantial after 30 years. Results are similar for the geographic constraint, not shown. Propositions 2 and 6 prove that the effect on rents is smaller than the effect on ownership prices, while Table 1 and Figure 1 illustrate that this difference is meaningful.

Population constraints decrease structure and lot sizes by 1.6%, holding income constant. To compute this number, we calculate the drop for each household in R after 30 years and then take the average across households. The 1.6% decrease in structure and lot sizes is nearly an

order of magnitude less than the increase in ownership prices and is significantly less than the increase in rents. Households pay for the permit price by cutting back on both housing and non-housing consumption. Because housing begins as only 25% of household budgets, the permit price does not reduce structure and lot size very much.

The effect of delays on the average housing characteristics in the city is not as negative as the effect given household income, as lower-income households are more likely to move out of city R (as predicted by Proposition 3). However, the effect is still negative and is close to zero. Consistent with sorting by income, population constraints raise the median income in the city by 3.0%. Finally, population constraints raise the expenditures for the average household remaining in R by 2.7%. This increase is smaller than the increase in quality-adjusted rent because households reduce their consumption of structure and land.

Geographic constraints reduce lot sizes by 18.8%, which is nearly double the effect on house prices. This type of constraint has no effect on structure size, housing expenditure shares, or median city income. The lack of adjustment along these dimensions results from Cobb-Douglas preferences and housing production, through which an increase in the unit price of land leads only to less land consumption and some out-migration.

The remaining columns of Table 1 explore the case of geographic constraints while relaxing the Cobb-Douglas assumptions. We instead use constant elasticity of substitution (CES) preferences or production, for which Cobb-Douglas is a special case. The appendix solves this more general model. Column (3) reports results when preferences are CES, in which case we take the elasticity of substitution between housing and non-housing consumption from Albouy, Ehrlich, and Liu (2016). In column (4), we also use a CES production function, taking the elasticity of substitution between land and structure from Albouy and Ehrlich (2018). In both cases, we keep the initial expenditure share on structure and housing the same as in the baseline calibration.

With CES preferences, households cut lot consumption by 13.9%, still a large number but less extreme than before. They pay for this smaller decline in lot size by reducing non-housing consumption. As a result, the expenditure share on housing rises 2.1%. While lot sizes still fall, structure sizes actually increase because, with a Cobb-Douglas production function, structure costs must scale with total housing costs. When the housing production function also is CES, lot sizes only fall by 6.6%. Housing expenditures still rise by 2.1%, but structure sizes now fall slightly by 1.4%. CES production makes structure and lots strong complements, meaning that developers cut structure sizes in response to the increase in land prices. In summary, lot sizes shrink markedly in these CES extensions but not by as much as in the Cobb-Douglas case. Effects on other outcomes remain small.

3. Empirical Strategy and Data

3.1. Identification

Our goal is to estimate the effect of housing supply constraints on housing affordability, as measured by rent, and on households' housing consumption and location decisions. We identify these effects by comparing outcomes across metropolitan areas in the US. Because of the large amount of heterogeneity in regulatory and geographic environments across locations, cross-metro analysis provides a good environment in which to look for its effects. One major empirical challenge, however, is that housing supply regulations correlate with many other aspects of local housing and labor markets that also affect the outcomes that we are interested in (Davidoff 2016). Therefore, we cannot simply regress our outcomes of interest on regulatory variables and expect to identify a causal effect.

We address this issue in three ways. The first way is to focus on changes in our outcomes of interest over time. This strategy allows us to abstract from other factors that might be correlated with regulation and housing characteristics, but are unchanging over time. For example, structure costs might vary across locations due to the availability of various types of construction materials. Or preferences over housing versus other consumption might differ across locations. The second way is to control for some time-varying factors that might be correlated with regulation and housing outcomes—specifically variables that reflect local productivity growth and amenities. The third way is to exclude metropolitan areas with low housing demand from our analysis, since housing markets in these areas likely differ from other areas in many unobservable ways that might be correlated with our outcomes of interest. We write this identification strategy as:

$$Y_{imt} = \delta_m + \delta_t + \beta_z Z_{mt} + \beta_x X_{imt} + \epsilon_{imt},$$

where Y_{imt} is an outcome for household i in metro m at time t , δ_m is a metro dummy, δ_t is a time dummy, Z_{mt} is a vector of supply constraints in metro m at time t , and X_{imt} is a vector of controls. The coefficient of interest is β_z .

We do not have detailed data on how supply constraints have changed over time, so we cannot include these changes directly in our analysis. Instead, we assume that these constraints have become more binding over the past four decades. Motivated by this assumption, we compare observations from 1980 ($t = old$) to observations in the 2010s ($t = recent$). Given that $Z_{m,old} = 0$, we may rewrite the above specification as

$$Y_{imt} = \delta_m + \delta_{t=recent} + \beta_z 1_{t=recent} Z_m + \beta_x 1_{t=recent} X_{im} + \epsilon_{imt},$$

where Z_m equals the average value of the supply constraint and X_{im} equals the average values of the controls that we use to proxy for changes in local productivity and amenities.

Prior research examining the evolution of housing supply regulation in specific locations supports the assumption that supply constraints were not very important before 1980. In a sample of 402 California cities, Jackson (2016) finds that most regulations that affect the size, location, or density of the housing stock were established after 1985. In a study of communities in the Greater Boston area, Glaeser and Ward (2009) show that most cluster zoning regulations, wetlands bylaws, and septic system requirements were adopted in the 1980s or later. While subdivision requirements were more common than these other regulations in the 1970s, nearly half of the communities in their sample adopted subdivision requirements after 1980. Massachusetts and California are well-known to be among the more highly-regulated states, so it is unlikely that housing supply regulations became widespread in other states before reaching these two states. Ganong and Shoag (2017) report a steady increase in the fraction of state appellate court cases that contain the phrase “land use” from 1980 to 2010—from about 0.25 percent in 1980 to 0.4 percent in 2010.⁴

We can look for further evidence of changes in the regulatory environment using data from two surveys conducted by researchers at the Wharton School of Business. Both surveys asked local government officials about the length of time typically required for a building permit application to be approved. The first survey was conducted in the 1980s (Linneman, Summers, Brooks and Buist 1990), and the second survey was conducted in 2006 (Gyourko, Saiz and Summers 2008). Table 2 reports the approval times for the 60 metropolitan areas that were covered in both surveys. In the 1980s, median application time for a single-family construction permit was 2 months, and the 90th percentile was 3 months. By contrast, in 2006 median application times ranged between 6 and 8 months, depending on the type of permit. And 90 percent of metropolitan areas had permit approval times greater than 3 months. The median increase in approval time across metro areas was in the range between 4 and 6 months. And because approval times were so low in all metro areas in the 1980s, the locations with the longest approval times in the 2006 are also the ones that experienced the largest increases in approval time.

The topography of the land changes quite slowly over time, so one might question how geographic constraints might become more binding over time. Cosman, Davidoff and Williams (2018) develop a model to show that in an expanding city, it is the marginal supply of land at the edge of the city that affects prices, not the average supply of land throughout the city. They argue that the marginal supply of land at the edge of the city does not decrease over time since the boundary of the city shifts out. However, in some metropolitan areas like San Francisco the terrain becomes more mountainous toward the edge of the metro, so the constraints become more binding as the metro grows toward these constraints. Moreover, infill development is

⁴ The incidence of court cases related to land use began increasing in 1960, illustrating that some regulations were binding in some locations prior to 1980.

fairly common in many metropolitan areas, and as housing demand in a city increases and more homes get built, less land will be available in the interior of the city for further new construction. To demonstrate the importance of infill development, Figure 2 shows how housing unit density in the central parts of metropolitan areas has changed from 1980 to 2016.⁵ In 1980, about two-thirds of metropolitan areas had an average density of less than 40 units per square kilometer in their central counties. By 2016, only about one-third of metros had an average density this low in their central counties. Thus, there has been a substantial amount of residential construction in the interior of metropolitan areas, and so we think it is reasonable to assume that the supply of land throughout the city matters for determining the supply of housing.

Our specification identifies β_z when $E(\epsilon_{imt} | Z_m, X_{im}) = 0$. The controls must explain all of the changes in the outcomes over time within metros that correlate with the growth in supply constraints but are not caused by the supply constraints. The controls that we think are most important are proxies for productivity growth and changes in the value of local amenities. Metros that have witnessed growth in regulatory constraints have also seen higher productivity growth (Saiz, 2010; Davidoff 2016), which could increase household income and alter equilibrium housing characteristics. Similarly, many supply-constrained metros are in locales commonly viewed as having desirable amenities. The amenity premium may have increased over time, perhaps because the aggregate population has become wealthier. We will discuss the variables that we use as proxies for changes in local productivity and amenities below.

Our third attempt to address the endogeneity of supply constraints is to exclude low-demand metropolitan areas from our analysis. These areas have quite different housing market dynamics from growing areas, and they are different along many unobservable dimensions as well as observable dimensions. Moreover, it is unlikely that supply constraints would bind in these areas. Following Gyourko, Mayer and Sinai (2013) and Charles, Hurst, and Notowidigdo (2018), we calculate housing demand in each metro area as the sum of the percent change in the number of housing units and the percent change in house value from 1980 to 2016.⁶ Low-demand areas are those in the bottom quartile of the demand distribution, and are dropped from our analysis. Figure 3 plots growth in the housing stock against growth in house values over this period and shows the dropped metro areas in blue.

3.2. Data on supply constraints

⁵ In the 2013 designation of which counties are in metropolitan areas, the Census Bureau identifies some counties in each metropolitan area as “central”. We use this indicator to define central counties and limit our analysis to metropolitan areas for which not all counties are designated as central.

⁶ Data are from the 1980 Census and 2016 American Community Survey. We take published data by county and aggregate to the 2013 metro area definitions. House value is calculated as the housing unit-weighted average of county median values.

As a proxy for constraints that reduce the future growth rate of the housing stock, we use an index of the strictness of housing supply regulation based on the Wharton Residential Land Use Regulation survey (Gyourko, Saiz and Summers 2008). In 2006, these researchers sent a survey to local government officials asking a range of questions about the types of residential land use regulation currently used and the political process through which land use regulations are formed. They combine the answers to the questions into a single index of regulatory stringency which is available for 259 metropolitan areas. The index is normalized to have a mean of zero and a standard deviation equal to one.

As a proxy for the supply of buildable land, we use data on geographic constraints.⁷ Specifically, we use the data underlying Saiz's (2010) estimates of the fraction of land that is unavailable for construction because it is on a steep slope or covered by water.⁸ This measure is also normalized to have a mean of zero and a standard deviation equal to one. The regulation index and the index of geographic constraints constitute our two components of Z_m .

Not only are the estimated effects of geographic constraints interesting in their own right, but they are helpful to include in our analysis for better identification of the effects of regulation. For example, Saiz (2010) shows that stricter housing supply regulations developed in areas with tighter geographic constraints. Also, the mountains and bodies of water that make it more difficult to build are frequently seen as positive amenities, and an increase in the desirability of these amenities from the 1970s to the 2000s may have raised housing demand in areas with tight geographic constraints (Cosman, Davidoff and Williams 2018). Consequently, while the identification strategy is not as clear for the geographic constraints as it is for the regulatory constraints, we would want to include the geographic constraints anyway in order to more clearly identify the effect of regulation.

3.3. Data on outcomes

To examine affordability directly, we use data on rent and property value from the 1980 Census and the 2012-2016 American Community Survey (ACS).⁹ Specifically, we use the variable reflecting gross rent, which adds utilities costs to contract rent in cases when utilities

⁷ One of the components of the regulation index is related to open space requirements, which one could view as a constraint on the supply of buildable land. However, it is so strongly correlated with other components of the index that we do not believe it is possible to use it to identify the effects of land supply separately from other types of regulation.

⁸ Saiz (2010) calculates these constraints for a radius of 50 kilometers around the center of each of 100 metropolitan areas. We alter this calculation slightly by calculating the fraction of unavailable land for all of the land area in the metropolitan area, which allows us to compute geographic constraints for a larger set of metropolitan areas.

⁹ Data obtained from the IPUMS USA (Ruggles et al. 2019). To harmonize the definition of metropolitan area over these two samples, we construct a cross-walk from the four-digit metropolitan delineations used in 1980 (IPUMS variable METAREA) to the 2013 OMB delineations (IPUMS variable MET2013).

are not already included, to ensure comparability across units. We assign a value of $t = old$ to all responses in the 1980 Census and a value of $t = new$ to all responses in the 2012-2016 ACS.

Our housing consumption outcomes come from two different sources of property-level data. The first is a 2014 cross-section of tax assessor data collected by CoreLogic. Tax assessors record a variety of property characteristics for the purpose of assessing property values and determining property taxes. This dataset covers the vast majority of single-family housing in the US, although important variables are missing or have unreasonable values in a non-trivial number of cases.¹⁰ Importantly for our purposes, we can obtain information on the square footage of the housing unit, the square footage of the lot, and the construction year of the property. We use the natural logarithm of unit size and lot size as outcomes. We assign a value of $t = old$ to any house built between 1960 and 1980 and a value of $t = new$ to any house built on or after 2000.¹¹ We drop units built before 1960 or between 1980 and 2000 from the analysis.

While the tax assessor data provide the most comprehensive data on housing unit characteristics with coverage across all metropolitan areas in the US, two drawbacks of the data are worth discussing. The first is that we only observe housing characteristics as they were in 2014. To the extent that some homes built in the 1960s and 1970s have been renovated, their 2014 characteristics do not reflect the characteristics at the time the homes were built. The second drawback is that the data only cover single-family homes. To the extent that household demand can switch between single-family and multifamily units in response to price changes, these data may not capture all of the effects we are interested in.

To address these drawbacks, we return to the Census/ACS data and examine two additional outcomes. The first outcome is an indicator for whether a property is a single-family structure. We interpret this outcome as a measure of lot size, since single-family homes are associated with much bigger lots (per housing unit) than multifamily homes. That said, single-family homes tend to be larger than multifamily units, so the single-family indicator should also be correlated with housing unit size.¹² The Census and ACS data do not have good measures of the size of the housing unit or lot.¹³ The second outcome that we examine is the number of adults

¹⁰ For computational reasons, we use a 25 percent random sample with 19 million usable observations. Thus the full dataset, with the same restrictions, would have about 75 million observations.

¹¹ To prevent our results from being driven by outliers with very high values, we drop housing units larger than 10,000 square feet and units with lots larger than 175,000 square feet (about 4 acres). We also drop units with extremely small recorded lots (less than 2000 square feet) and units with very high ratios of floor area to lot size.

¹² In the 2015 American Housing Survey, the median size of single-family detached homes was 1805 square feet, while the median size of units in structures with 2 to 4 units was 900 square feet and the median size of units in structures with 50 or more units was 800 square feet. Table created at the AHS website: <https://www.census.gov/programs-surveys/ahs/data/interactive/ahstablecreator.html>.

¹³ The datasets do record the number of rooms and number of bedrooms. However, the tax assessor data show only a weak correlation between the number of rooms or bedrooms and the actual size of housing units,

per household, since people living in larger households generally consume less structure per person.

In order to examine the effects of housing supply constraints on sorting across metropolitan areas, we aggregate the Census/ACS data to the metro level and the outcome of interest becomes the change in a metropolitan area characteristic from 1980 to 2012-2016. The first set of characteristics that we examine are the fraction of people in each decile of the national distribution of income. Then, because annual income may not always reflect a person's permanent income, we also look at two proxies for permanent income: education and occupation. Specifically, we examine the fraction of the population age 25 and older with at least 4 years of college and the fraction of the population with a high occupation score. The occupation score is created by the Census Bureau using median incomes by detailed occupation category using the 1950 Census.

Finally, we examine the effects of the supply constraints on housing expenditures and the ratio of expenditures to household income. Such measures are frequently used in analyses of housing affordability. While our model clearly demonstrates that housing expenditures are not a good measure of affordability because they reflect household choices as well as the cost of housing services, we think these results provide a nice way to combine the effects on housing costs, housing consumption and location decisions.

3.4. Data on controls

Our empirical specification includes two proxies for productivity growth: the share of the population age 25 or older with at least 4 years of college in 1980, and the share of employment in industries that experienced fast wage growth from 1990 to 2016. Educational attainment is obtained from the 1980 Census.

To calculate the share of employment in high wage-growth industries, we calculate wage growth from 1990 to 2016 by industry using the annual files of the Quarterly Census of Employment and Wages (QCEW). Wages are defined as total annual wages divided by total annual employment. We define industries using 3-digit NAICS codes, which gives us 96 industry categories. Although we would prefer to calculate wage growth from 1980 to 2016, the QCEW data by NAICS industry are not available prior to 1990.¹⁴ We define high wage growth

likely because larger homes tend to have larger rooms, not just more rooms. Another indicator that the number of rooms is a poor proxy for housing unit size is that there is little variation in the average or median number of rooms across metropolitan areas in the Census/ACS data.

¹⁴ We could use data by 1-digit SIC code to extend our analysis back to 1980. However, doing so would give us only about 10 industry categories, and we think having more detailed industry definitions is more valuable than having a longer time period.

industries as those in the top decile of wage growth and calculate the fraction of employment in 1990 in those industries.

We use three proxies for local amenities. The first is average January temperature. The value of nice weather seems to have increased since the 1970s (Glaeser and Gyourko, 2003) and many supply-constrained metros are in warm locales such as California. This weather premium may have affected land prices, and hence housing characteristics, in ways we do not want to attribute to supply constraints. We obtain average January temperature by weather station from 1981 to 2010 from the National Oceanic and Atmospheric Administration. We average the station-level data by county, then take a weighted average across counties within each metropolitan area using county land area as weights.

The second proxy for local amenities is the fraction of employment in 1980 that is related to the provision of local consumption amenities. As incomes have risen over time, the value of consumption amenities has increased (Couture and Handbury 2019, Glaeser, Kolko and Saiz 2001). We define industries as providing local consumption amenities if they are in the following SIC-based industries: eating and drinking places, amusement and recreation services, and museums and zoos. Because these industries are based on SIC codes, we are able to calculate their employment shares in the QCEW data as of 1980.

The third proxy for local amenities is the share of seasonally-vacant housing from the 1980 Census. Demand for seasonal housing has grown over time with the ageing of the population and rising incomes, and seasonal housing tends to be in high-amenity areas that also may have tighter topographic or regulatory constraints. To make coefficients comparable across variables, we standardize all five of the controls to have a mean equal to zero and standard deviation equal to one. These measures are generally positively correlated with growth in the housing stock from 1980 to 2016, consistent with the interpretation that they reflect strong housing demand (see Appendix Table 1).¹⁵

Beyond the metro-level controls for productivity and amenities, two other sets of controls bear mentioning. For the specifications with rent and house value as the dependent variable, we control for all available property characteristics: building age, number of rooms, number of bedrooms and a single-family indicator.¹⁶ The reason for these controls is because we are interested in constant-quality rent and price effects.

¹⁵ The fraction of highly-educated adults is roughly uncorrelated with housing stock growth in the regressions shown in the Appendix. However, we prefer to include this variable as a control because it is a common proxy for local productivity.

¹⁶ Specifically, we include indicators for decade of year built, indicators for each value of number of bedrooms, and indicators for each value of number of rooms.

For specifications that examine housing characteristics as an outcome, we need to control for household income. As shown by the model, doing so accounts for the effects of supply constraints on sorting across metros, isolating the effects on the choices made by households of a given income level. The specific method of controlling for household income depends on the outcome data we are using. When we are using the Census and ACS data, we include indicators for the household's decile in the national distribution of household income. We allow for this flexible specification of income in case housing consumption choices are not a linear function of income. When we are using the tax assessor data, we control for interactions of the time period indicator with decile in the national income distribution, using median Census tract income from the 1980 Census.¹⁷

4. Results

4.1. *Effects on Housing Affordability*

We start by examining the effects of housing supply constraints on real house prices and rents. The first column of Table 3 reports the estimated effects of our two supply constraints on single-family house values in the Census/ACS data. A metropolitan area with regulations that are one standard deviation tighter than average experienced a 0.094 log point (about 10 percent) stronger house price appreciation over our sample period. The estimated effect of geographic constraints is slightly larger. Results are similar when we measure house prices using a repeat-sales price index instead of owner-reported house values in the Census/ACS (not shown).

Effects of this magnitude are considerable. To illustrate, we convert the coefficient estimates to growth rates for a metropolitan area with average supply constraints (using the regression constant) and for a highly-regulated metropolitan area that has a constraint two standard deviations above the mean. The bottom rows of the table show that house prices doubled (in real terms) in a highly regulated area, whereas they rose by 63 percent in a metropolitan area with average regulation. Even so, it is worth noting that our identification strategy leads us to estimate much smaller effects than one would expect from the raw correlations in the data. For example, the estimated effect of regulation would be more than three times as large if we were to estimate it from the cross section of metropolitan areas in 2016 with no other controls. Estimation based on changes in house values from 1980 to 2016 reduces the estimated effect by about half, and the estimated effect is reduced further by

¹⁷ Specifically, we use 1980 Census data imputed to 2010 Census tracts from Logan, Xu and Stults (2014).

controlling for geographic constraints, controlling for productivity and amenities, and excluding low-demand areas (see Appendix Table 1).

The second column of Table 3 reports the estimated effects of supply constraints on the rent of single-family homes. We start with single-family rentals because these structures are more similar to the structures used to estimate the effects on house prices. For each supply constraint, the estimated effect on rent growth is less than half of the estimated effect on house price growth. The third column of Table 3 reports the estimated effects on rent in a sample of all rental homes, which is a more comprehensive sample of rental units. Still the estimated effects on rent are less than half as large as the estimated effect on prices. These results are especially striking because the average increase in real rent over this period was about the same as the average increase in real house prices, as shown by the coefficients on the 2012-2016 indicators. Thus, consistent with the model, we find that supply constraints increase rent by much less than house prices.

Not only are the estimated effects on rent small relative to the effect on prices, they are small in absolute magnitude. For example, based on column 3 a metro area with regulation 2 standard deviations tighter than average experienced only 0.07 log point larger rent increases from 1980 to 2016, which works out to less than $\frac{1}{4}$ percentage point faster growth per year. The fourth column of Table 3 reports the estimated effects of supply constraints on the rent of 2-bedroom apartments, which is a structure type commonly occupied by low-income households. While the magnitudes are a bit larger for this sample than for the sample of all rental units, they still suggest that metropolitan areas with 2 standard deviation tighter regulatory constraints than average experienced only a 0.094 log point larger increase in real rent growth over this 36 year period (about $\frac{1}{4}$ percentage point faster growth per year). Thus, we find that supply constraints have only reduced housing affordability by a modest amount over this period.

One immediate question that may come to mind is whether our measures of supply constraints may be poor proxies for true supply constraints, which would cause us to underestimate the effects on prices and rent. While there is surely some degree of measurement error in these measures, they are commonly-used in academic research and have been shown to be correlated with the elasticity of housing supply (Saiz 2010).¹⁸ One way to assess our estimated effects is to compare them to other research that identifies these effects using other data and other identification strategies. Few academic studies have estimated causal effects of supply constraints on house prices owing to the difficulties with measurement and endogeneity (Gyourko and Molloy 2015). The most comparable analysis we have found is

¹⁸ To date, the papers introducing the regulatory index and the geographic constraint measure have been cited in 170 and 347 published journal articles, respectively.

Hilber and Vermeulen (2016), who examine the effects of supply constraints on house prices across local planning authorities in England, instrumenting for regulation using a policy reform and spatial variation in Labour party votes. They find that a one standard deviation decrease in regulation is associated with 14 percentage point lower cumulative house price growth from 1974 to 2008, a result that is in line with our estimated effects on house price growth in the US.

Another question that may come to mind is whether rent control might prevent rents from responding to supply constraints as much as prices. We obtain a list of jurisdictions with rent control in 2014 from Landlord.com¹⁹ and drop metropolitan areas with any jurisdictions that have rent control.²⁰ The estimated effects on rent in this sample remain about half of the estimated effect on house prices, indicating that rent control cannot explain the differential between these two outcomes (see Appendix Table 3).

A third concern with our analysis is that the rents paid by tenants may not reflect market rents if the tenants have occupied the unit for a long time. We address this issue by limiting our sample to households where the household head moved in within the previous 5 years. Again, we find estimated effects on rent that are only half as big as the estimated effects on house prices (see Appendix Table 4).

4.2. Effects on Housing Consumption

One might be skeptical of our ability to directly examine the effects of supply constraints on the price of housing services because this concept is impossible to observe for owner-occupied housing, which makes up roughly two thirds of all housing units. The model illustrates how an increase in the price of housing services should cause households with a given income to reduce their consumption of housing services. In the case of population constraints, this showed up as small decreases in structure size and lot size. In the case of land area constraints, this showed up as a fairly sizeable decrease in lot size, with small changes in structure size depending on the elasticities of substitution between lots and structure and between housing and non-housing consumption. Therefore, next we examine the effects of housing supply constraints on direct measures of housing consumption.

The first three columns of table 4 reports the estimated effects on structure size and lot size of single-family homes, and also on a single-family indicator that signals both a larger structure and a larger lot. The coefficients on regulation and geographic constraints are all small in magnitude and insignificantly different from zero. In some cases, they are positive instead of negative as expected. We can reject that a 1 standard deviation increase in regulatory

¹⁹ http://www.landlord.com/rent_control_laws_by_state.htm.

²⁰ There are 13 metropolitan areas with rent control according to this definition. We treat the metro areas of Washington DC and Riverside CA as having rent control, even though most jurisdictions in these metros do not have rent control. Results are similar if we treat these two metros as not having rent control.

constraints reduces structure size or lot size by more than 1½ percent, which was the magnitude of the effect predicted by the model. We can also reject that a 1 standard deviation increase in geographic constraints reduces lot size by more than 6 percent, the smallest of the range of effects predicted by the model. These results seem inconsistent with the model, and are especially surprising given that the estimated effects on prices and rents were in line with the model simulation.

One interpretation of these empirical results is that supply constraints have a smaller effect on the price of housing services than predicted by the model, and also have smaller effects than we estimate using rent data. Another possibility is that people generally adapt to the higher price of housing services in ways that are not directly captured by our model. One such method of adapting could be through changes in household size. In the model, household size is fixed and we can think of the predictions for housing consumption as consumption per person. But in reality household sizes do vary considerably and could depend on housing costs.

We examine empirical effects on housing consumption per person in the Census and ACS data. Using individual-level data, we estimate the effects of our two supply constraints on the logarithm of the inverse of the number of adults (which we define as age 18 or over) in that individual's household. This outcome can be thought of as reflecting the amount of structure consumed by that person. We control for income by dividing household income by the number of adults in the household and calculating their decile in the national income distribution.²¹

The last column of table 4 shows a statistically significant negative effect of each supply constraint on structure per adult. A one standard deviation greater degree of regulation is estimated to reduce the amount of structure per adult by about 1 percent, a magnitude that is in line with the model's predicted effects on structure size. This result suggests that instead of living in smaller homes, people choose to reduce their consumption of structures by living in a larger household. The estimated effect of geographic constraints is also negative and a bit larger in magnitude, with a one standard deviation greater degree of regulation reducing structure per person by 2½ percent. This result is roughly consistent with the version of the model in which the elasticity of substitution between lot and structure is less than one.

Another possible explanation for these empirical estimates is that other aspects of the housing market or regulatory environment might prevent people from altering their housing consumption decisions as much as predicted by the model. For example, regulatory constraints are more common in metro areas with tighter geographic constraints, and some of these

²¹ We do not control for individual income because it may not accurately represent purchasing power among couples in which one individual does not work. In a separate robustness check, we have also confirmed that demographic changes are not driving these results by also controlling for the interactions of age, sex, and year dummy variables.

regulations constrain the size and shape of lots. Another factor could be the durability of housing, which means that the existing housing stock will slow to adapt to changes in the regulatory environment and changes in housing demand. We investigate this possibility by estimating the effects of supply constraints on the single-family indicator in a sample of homes built within the previous 10 years.²² Although the estimated effect of regulatory constraints is unchanged, the estimated effect of geographic constraints becomes more negative (see Appendix Table 5). Therefore, it seems plausible that other constraints may be preventing households from fully adapting their housing consumption choices to changes in housing affordability.

A final explanation that we consider is the idea that home owners experience wealth gains from appreciation in house values, and they can use this additional housing wealth to increase their housing consumption. In support of this possibility, the effect of geographic constraints on the single-family indicator is more negative for renters than it is for home owners (see Appendix Table 5). It is also more negative for young owners, who have had less time to build up housing equity, than it is for older owners (see Appendix Table 5). Thus, wealth gains by owners of real estate may be mitigating the need to reduce housing consumption in response to decreases in housing affordability.

4.3. Effects on Sorting Across Metropolitan Areas

Next we examine the effects of housing supply constraints on sorting across metropolitan areas. The model predicted that population growth would be lower in areas with greater supply constraints. Prior research has generally found regulatory constraints to reduce growth in the housing stock (Mayer and Somerville 2000, Saks 2008, Jackson 2016). There has been less research on the effects of geographic constraints on local housing or population growth, and the research on the effects of regulation generally does not control for geographic constraints. Consequently, we start by estimating effects on the housing stock using our data and identification strategy.

Table 5 reports the results of regressing the change in a metro's housing stock from 1980 to 2016 on our two supply constraints, controlling for metro area productivity and amenities. We find a fairly sizeable negative effect of geographic constraints on the local housing stock—a one standard deviation increase in geographic constraints is associated with 8 percentage point lower housing stock growth. This magnitude is larger than the model's predicted effect of land constraints on population growth, which was only about 2 percent.

²² We cannot undertake a similar exercise for the unit size or lot size data because we only observe these homes in 2014.

The estimated effect of regulatory constraints is also negative, but is much smaller and insignificantly different from zero. One factor contributing to this result is the positive correlation between regulatory and geographic constraints. If we exclude the geographic constraints from the regression, the estimated effect of regulatory constraints doubles in magnitude (see Appendix Table 2). In specifications with fewer controls, we generally find a positive relationship between regulation and housing stock growth (see Appendix Table 2). This result illustrates the positive correlation between regulations and a variety of local factors that increase local housing demand, highlighting the importance of controlling for these factors when attempting to estimate the effect of regulations on housing outcomes. While unobserved factors boosting local demand should bias our estimated effects of regulation on the housing stock downward, they should bias our estimated effects of regulation on house values and house prices upward, meaning that the true effects of regulation on housing affordability may be even smaller than the small effects that we estimate.

Next, we turn to how supply constraints affect the types of people who choose to live in the area. The model predicted that people with more income would be more likely to stay in areas with population constraints, while it predicted no effect of land supply constraints on sorting because we assumed that an individual's taste for the regulated city is uncorrelated with income. If instead we were to assume that income is positively correlated with changes in the taste for the regulated city—say because the regulated city has amenities that have become more valued by richer people—then we would expect land supply constraints also to cause sorting by income.

We first look for evidence of income-based sorting using data from the Census and ACS on income. Specifically, we calculate the fraction of individuals in a metropolitan area that are in each decile of the national distribution of income. An increase in the fraction of individuals in the upper deciles would be consistent with richer people sorting into that metropolitan area. Therefore, we regress the change in the fraction of individuals in a decile on the supply constraints and metro-level controls for productivity and amenities.

Figure 4 plots the coefficient estimates for each decile. The results are consistent with a mild amount of sorting in response to regulatory constraints, as these constraints have led to larger shares of individuals in the top two deciles and a smaller share of individuals in lower deciles (although only the 4th decile is significantly different from zero). But these effects are not large, as a 1 standard deviation greater regulatory constraint is associated with only about ½ percent more of the population being in each of the top two deciles. Similarly, we find that a 1 standard deviation increase in regulatory constraints is associated with only a 2 percent increase in real median income (Table 5). This small magnitude is consistent with the magnitude implied by the model.

Figure 4 and Table 5 show no evidence of income sorting across metropolitan areas in response to geographic constraints, consistent with a model in which preferences for local amenities are uncorrelated with income.

Next we look at effects on sorting by education and occupation. Regulation is associated with a small increase in the fraction of highly-educated adults. The estimated effect on the fraction of people in high-income occupations is small and insignificantly different from zero. As with the income results, geographic constraints are unrelated to these measures of permanent income.

To get a sense of the magnitudes of these effects, consider the metropolitan area of San Francisco, which has an appreciable amount of regulation and experienced large increases in its fractions of high-income and highly-educated residents from 1980 to 2016. Our estimated coefficients imply that regulatory constraints can only explain one fifth of the increase in the share of residents in the top decile of the income distribution and one twentieth of the increase in the share of highly-educated residents.

As a final method of gauging the amount of income sorting across metropolitan areas, we estimate effects of supply constraints on housing consumption without controlling for income. If supply constraints were causing a substantial amount of sorting by income, we would expect to find effects on average housing characteristics that are less negative than the estimated effects on households with a given income. Indeed, the model simulation predicts that average structure size and lot size will be higher in more regulated cities, even though structure and lot consumption is lower at each level of income. However, when we omit the income controls from each of our housing consumption regressions, in no case do the estimated effects on an outcome to become materially less negative (see Figure 5). On net, our results suggest that much of the sorting across metropolitan areas that has occurred is due to factors other than constraints on the housing supply.

4.4. Effects on Sorting Within Metropolitan Areas

Another set of outcomes related to location choice that we examine is location within metropolitan areas. Our model did not differentiate across locations within metropolitan areas, so it does not make any predictions for this type of sorting. However, it is easy to imagine that households might also adjust to higher land prices by choosing to live in a relatively cheaper neighborhood within the metro area.

We assess this possibility by examining whether new housing units are more likely to be located in less-desirable neighborhoods in metropolitan areas with tighter housing supply constraints. Neighborhood desirability is measured using four separate neighborhood

characteristics (where neighborhoods are defined as Census tracts): distance to the central business district (CBD), average commute time, crime, and school quality. The center of the metropolitan area comes from Holian and Kahn (2015).²³ Commute time is measured in the 2011-2015 ACS. School quality data are obtained from Location Inc., and are derived by adjusting local test score data across states using nationwide test scores to make scores comparable across school districts. Crime rate data are also obtained from Location Inc., and are calculated by assigning crimes reported by all law enforcement agencies in the U.S. to Census tracts using a proprietary model. The education and crime variables range from 0 to 100, representing the percentile in the national distribution.

We estimate the effect of supply constraints on location choice within the metro in the CoreLogic property tax data by regressing each of the four neighborhood characteristics on an indicator for whether the home was built post-2000 and an interaction of this indicator with each supply constraint. The regression controls for metropolitan area fixed effects, neighborhood income, and metro-level productivity and amenities also interacted with the “post-2000” indicator. This specification thus reveals whether homes built post-2000 were more likely to be in lower-amenity neighborhoods if they are in more supply constrained metro areas, relative to the distribution of housing units in the 1960s and 1970s.

Table 6 reports the results. Perhaps not surprisingly, recently built homes are more likely to be located farther from the CBD, and in areas with longer average commutes, better schools, and less crime. We find no statistically significant or economically meaningful interactions between these effects and regulatory constraints; that is, new homes in more regulated areas are built in similar areas to those in less regulated areas. In contrast, geographic constraints do seem to have significant effects on location choice. Relative to less constrained metros, those with greater geographic constraints tend to have newer homes closer to the CBD and with lower commute times, as well as in areas with lower school quality and higher crime. We find similar results when we look for these effects by regressing tract-level population growth on the four neighborhood quality measures and interact these measures with our supply constraints (see Appendix Table 6). While these interactions with geographic constraints are significant, even in the tail of the distribution—for example, two standard deviations above the mean on the index of geographic constraints—new homes are still more likely than existing homes to be built farther from the CBD and in areas with longer commutes, lower school quality, and higher crime. That is, the interaction terms do not overwhelm the main effects in the first row of the table.

²³ Holian and Kahn (2015) use the location returned when entering the central city name in Google Earth, which they found to be qualitatively “quite reasonable in all cases”. The data are available for download at <http://mattholian.blogspot.com/2013/05/central-business-district-geocodes.html>.

One important caveat to this analysis is that we cannot observe variation in supply constraints across neighborhoods. If supply constraints were tighter in less-desirable neighborhoods, households would be less likely to choose these neighborhoods, possibly offsetting the effect that we expected. On the other hand, most research has found that regulations are more likely to be found in wealthier areas with more desirable amenities (Davidoff 2016). This would create an additional mechanism by which regulation would push households into less desirable neighborhoods.

4.5. Effects on Housing Expenditures

Finally, we estimate the effects of the supply constraints on housing expenditures using the Census/ACS data at the household level. Expenditures are measured as rent for renter households, and monthly payments for owner-occupied households (which includes mortgage interest, property taxes and homeowners insurance). Other than including data on owner expenditures, the main difference between this specification and the specification estimating effects on quality-adjusted rent (reported in Table 3) is that we do not control for housing unit characteristics. Thus we are estimating effects on rental and owner expenditures instead of constant-quality rent or house values.

Table 7 reports the coefficient estimates. A one standard deviation tighter degree of regulation is associated with 4 percent higher expenditures, an effect quite similar to the prediction of our model. A one standard deviation tighter degree of geographic constraint is associated with 5 percent higher expenditures. The Cobb-Douglass version of the model predicted that housing expenditures would not respond to a decrease in land supply, but allowing for an inelastic substitution between housing and non-housing causes the model to predict a 2 percent increase in housing expenditures. As we discussed above, it is possible that other factors, such as minimum lot sizes or density restrictions, prevent people from fully offsetting the effects of a tighter land supply with less land consumption, which would in turn lead to higher housing expenditures.

The table also reports results where the dependent variable is an indicator for whether a household spends more than 30 percent of their income on housing, a common measure of housing “cost burden” in the affordability literature. Both constraints increase the fraction of “cost burdened” households by a small amount, but even the effects of a 2 standard deviation tighter constraint are much smaller than the increase in “cost burdened” households over our sample period.

It is possible that the housing wealth effects discussed above have allowed households to spend a greater fraction of their income on housing than they would have otherwise. Table 7 reports estimates for samples of owners and renters. The estimated effects for owners are a

little larger, but the only case where the difference appears material is for geographic constraints and $\ln(\text{expenditures})$).

We end this section with an examination of how the effects on housing expenditures vary with household income. Most of the concerns about housing affordability are aimed at lower income households, so it would be helpful to know if housing supply constraints have different effects for households at different points in the income distribution. Toward this end, we regress $\ln(\text{expenditures})$ and an indicator for expenditures exceeding 30 percent of income on our supply constraints, running separate regressions for each decile of the national income distribution.

Figure 6 shows the results. The estimated effects on expenditures are fairly similar across the income distribution, but are somewhat smaller for the lowest two deciles. The effects on the indicator for having high housing expenditures relative to income are more hump-shaped, with the largest effects of regulatory constraints in the 4th and 5th deciles and the largest effects of geographic constraints in the 5th and 6th deciles. It is possible that housing programs are helping to reduce the effects of supply constraints on housing expenditures for low income households. Meanwhile, because the effects on expenditures are fairly similar for middle- and high-income households, they are a smaller share of income as income rises.

5. Conclusion

We have shown both theoretically and empirically that housing supply constraints have a smaller effect on housing affordability than on the purchase price of housing. Supply constraints also have only limited effects on housing consumption and location decisions.

Our results may seem quite surprising in light of the strong cross-sectional correlation between supply constraints and rents. Indeed, even in our sample, the cross-sectional correlation between supply constraints and rents is three times larger than our estimated causal effects. It turns out that locations with tight supply constraints tended to have higher rent even back in 1980, so the changes in rent over time are not as strongly correlated with supply constraints as the current levels may suggest. Controlling for measurable differences in demand further reduces the estimated effects of supply constraints, suggesting that supply constraints are also correlated with strong housing demand.

One should not conclude from our analysis that housing affordability is not a problem in supply constrained metropolitan areas. Rather, our results suggest that the supply constraints alone have not been the driving force behind high rents. Why are our estimated causal effects so much smaller than the effects suggested by the cross-sectional correlation between rent and supply constraints? One possibility is that our measures of supply constraints are not good

proxies for true supply constraints. But our supply constraints *are* more strongly correlated with house price growth. Furthermore, these two constraints are the most commonly-used measures in the literature.

A second possibility is that supply constraints were at least somewhat binding even back in 1980, in which case we have underestimated the true effects of these constraints. Other research has documented the existence of some regulations prior to 1980—for example, Ganong and Shoag (2017) document the appearance of the words “land use” in state court cases as far back as 1950—and some geographic constraints were surely binding back then. But as we have shown, these constraints did become much stronger between 1980 and the 2000s.

A third possibility is that rent growth is more of a function of strong demand than of tight supply. Our proxies for demand shocks do not explain a large amount of the variation in rent growth across metropolitan areas, but they are only rough proxies for local demand. While it seems quite plausible that demand shocks could have played an important role, it is difficult to understand how rents could increase so much without some kind of limit on the housing supply. In our view, the most likely explanation is that high rents can be explained by a combination of strong demand and limited supply, as in Gyourko, Mayer and Sinai (2013). Therefore, it is important for researchers and policy makers to consider the confluence of demand and supply conditions when trying to understand why housing is so unaffordable in some areas and what to do about it.

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Table 1
Reponses to Supply Constraints that Raise Prices 10% over 30 Years
(Model Simulation, %)

	Population constraints	Land area constraints		
	(1)	(2)	(3)	(4)
Quality-adjusted rent (median)	6.0	5.9	5.9	5.7
Housing expenditure, city average	2.7	0.0	2.1	2.1
Structure size, holding income constant	-1.6	0.0	2.1	-1.4
Lot size, holding income constant	-1.6	-18.8	-13.9	-6.6
Structure size, city average	1.5	0.0	2.1	-1.4
Lot size, city average	1.5	-18.8	-13.9	-6.6
Median city income	3.0	0.0	0.0	0.0
Population	-3.6	-2.2	-2.2	-2.1
Housing services consumption	-2.1	-7.6	-5.6	-5.4
<i>Assumptions</i>				
Housing/non-housing substitution elasticity	-	1	0.5	0.5
Lot/structure substitution elasticity	-	1	1	0.33

Table 2
Months from Application to Permit Issuance for SF construction

Percentile	Application for Rezoning		Application for Subdivision	
	< 50 units	≥ 50 units	< 50 units	≥ 50 units
	1980s			
10 th	1	2	1	1
50 th	2	2	2	2
90 th	3	3	3	2
	2006			
10 th	3.9	4.8	3.6	3.8
50 th	6.4	8.0	5.6	6.8
90 th	10.8	13.0	9.0	10.7
	Change from 1980s to 2006			
10 th	1.7	2.4	2.2	2.6
50 th	4.7	5.8	4.1	5.2
90 th	8.0	10.2	6.7	8.7

Note. Sample includes the 60 metropolitan areas that appear in both surveys. Data from the 1980s are from a survey conducted by Linneman, Summers, Brooks and Buist (1990) and data from 2006 are from a survey conducted by Gyourko, Saiz and Summers (2008).

Table 3
Effect of Housing Supply Constraints on House Prices and Rent

	Ln(Value) SF Homes	Ln(Rent) SF homes	Ln(Rent) All homes	Ln(Rent) 2-Bed Apt.
2012-2016 Indicator	0.489 (0.023)	0.493 (0.015)	0.499 (0.013)	0.467 (0.015)
Indicator interacted with:				
Regulatory constraints	0.094 (0.023)	0.036 (0.011)	0.035 (0.010)	0.047 (0.012)
Geographic constraints	0.116 (0.022)	0.022 (0.011)	0.047 (0.011)	0.052 (0.013)
Controls for Housing Characteristics	Yes	Yes	Yes	Yes
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes
Outcome Data	Census/ACS	Census/ACS	Census/ACS	Census/ACS
Number of Observations	2.2 million	0.4 million	1.2 million	0.35 million
Model predictions: Increase in dependent variable 1980 to 2016 for a metro with:				
Average constraints	63%	64%	65%	60%
+ 2-SD regulation	97%	76%	77%	75%
+ 2-SD geog. Const.	106%	71%	81%	77%

Note. Standard errors are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. Controls for housing characteristics are indicators for decade built, indicators for number of rooms, and indicators for number of bedrooms. Value and rent are expressed relative to the price index for personal consumption expenditures. Controls for productivity and amenities are the following variables interacted with the “recent” indicator: fraction of population with 4+ years college in 1980, fraction of employment in high-wage employment in 1990, average January temperature 1980-2010, fraction of employment in consumption-related industries in 1980, and fraction seasonal housing units in 1980.

Table 4
Effect of Housing Supply Constraints on Housing Consumption

	Ln(Unit Size) SF Homes	Ln(Lot Size) SF Homes	SF indicator	Ln(Structure Per Adult)
“Recent” Indicator	0.380 (0.020)	-0.148 (0.035)	0.107 (0.007)	0.002 (0.006)
Indicator interacted with:				
Regulatory constraints	0.020 (0.011)	0.035 (0.023)	0.006 (0.006)	-0.013 (0.004)
Geographic constraints	-0.010 (0.011)	-0.027 (0.029)	-0.010 (0.006)	-0.024 (0.005)
Controls for Income	Yes	Yes	Yes	Yes
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes
Outcome Data	CoreLogic	CoreLogic	Census/ACS	Census/ACS
Number of Observations	4.2 million	4.2 million	3.7 million	7.5 million

Note. Standard errors are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. Controls for income are indicators for deciles in the national distribution of income and interactions of these indicators with the “recent” indicator. When the outcome uses Corelogic data, income is median household income by Census tract. When the outcome uses Census/ACS data, income is property-level household income. Controls for productivity and amenities are the following variables interacted with the “recent” indicator: fraction of population with 4+ years college in 1980, fraction of employment in high-wage employment in 1990, average January temperature 1980-2010, fraction of employment in consumption-related industries in 1980, and fraction seasonal housing units in 1980.

Table 5
Effect of Housing Supply Constraints on Changes in Metropolitan Area Housing Stock and Population Characteristics 1980 to 2016

	Ln(Housing Stock)	Fraction 4+ Years College	Fraction High Occupation Score	Ln(Median Income)
Constant	0.532 (0.019)	0.126 (0.004)	0.032 (0.002)	0.309 (0.011)
Regulatory Constraints	-0.016 (0.019)	0.009 (0.004)	0.003 (0.002)	0.018 (0.011)
Geographic Constraints	-0.082 (0.018)	-0.004 (0.004)	-0.003 (0.002)	-0.019 (0.010)
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes
Number of Observations	133	133	133	133

Note. The housing stock includes single-family and multifamily units. High occupation score is defined as above the 90th percentile of the national distribution of occupation scores in the same year. Controls for productivity and amenities are: fraction of population with 4+ years college in 1980, fraction of employment in high-wage employment in 1990, average January temperature 1980-2010, fraction of employment in consumption-related industries in 1980, and fraction seasonal housing units in 1980.

Table 6
Effect of Housing Supply Constraints on Neighborhood Choice

	Ln(Distance to Metro Center)	Ln(Average Commute Time)	Education Index	Crime Index
“Recent” Indicator	0.273 (0.032)	0.086 (0.008)	0.354 (0.039)	-0.553 (0.044)
Indicator interacted with:				
Regulatory Constraints	-0.010 (0.022)	0.009 (0.005)	-0.033 (0.021)	-0.018 (0.026)
Geographic Constraints	-0.092 (0.017)	-0.021 (0.005)	-0.131 (0.021)	0.170 (0.022)
Controls for Income	Yes	Yes	Yes	Yes
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes
Outcome Data	CoreLogic	CoreLogic	CoreLogic	CoreLogic
Number of Observations	4.2 million	4.2 million	4.2 million	4.2 million

Note. Standard errors are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. The education index and crime index are also standardized. Controls for income are median household income by Census tract interacted with decade indicators. Controls for productivity and amenities are the following variables interacted with the “recent” indicator: fraction of population with 4+ years college in 1980, fraction of employment in high-wage employment in 1990, average January temperature 1980-2010, fraction of employment in consumption-related industries in 1980, and fraction seasonal housing units in 1980.

Table 7
Effect of Housing Supply Constraints on Housing Expenditures

	Ln(Real Expenditure)			Expenditure > 30% of Income		
	Full Sample	Owners	Renters	Full Sample	Owners	Renters
“Recent” Indicator	0.409 (0.013)	0.370 (0.015)	0.458 (0.014)	0.139 (0.014)	0.102 (0.008)	0.192 (0.021)
Indicator interacted with:						
Regulatory constraints	0.043 (0.009)	0.041 (0.011)	0.033 (0.009)	0.026 (0.004)	0.024 (0.004)	0.020 (0.005)
Geographic constraints	0.051 (0.009)	0.064 (0.012)	0.029 (0.009)	0.021 (0.004)	0.022 (0.004)	0.017 (0.005)
Controls for Income	Yes	Yes	Yes	Yes	Yes	Yes
Control for metro area productivity and amenities	Yes	Yes	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Number of Observations	3.6 mil.	2.4 mil.	1.2 mil.	3.7 mil.	2.4 mil.	1.2 mil.

Note. Standard errors are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. Controls for income are indicators for the household’s decile in the national distribution of household income and interactions of these indicators with the “recent” indicator. Controls for productivity and amenities are the following variables interacted with the “recent” indicator: fraction of population with 4+ years college in 1980, fraction of employment in high-wage employment in 1990, average January temperature 1980-2010, fraction of employment in consumption-related industries in 1980, and fraction seasonal housing units in 1980. Expenditures are deflated by the price index for personal consumption expenditures.

Figure 1
Increases in House Prices and Rent in Response to an Unanticipated Population Constraint

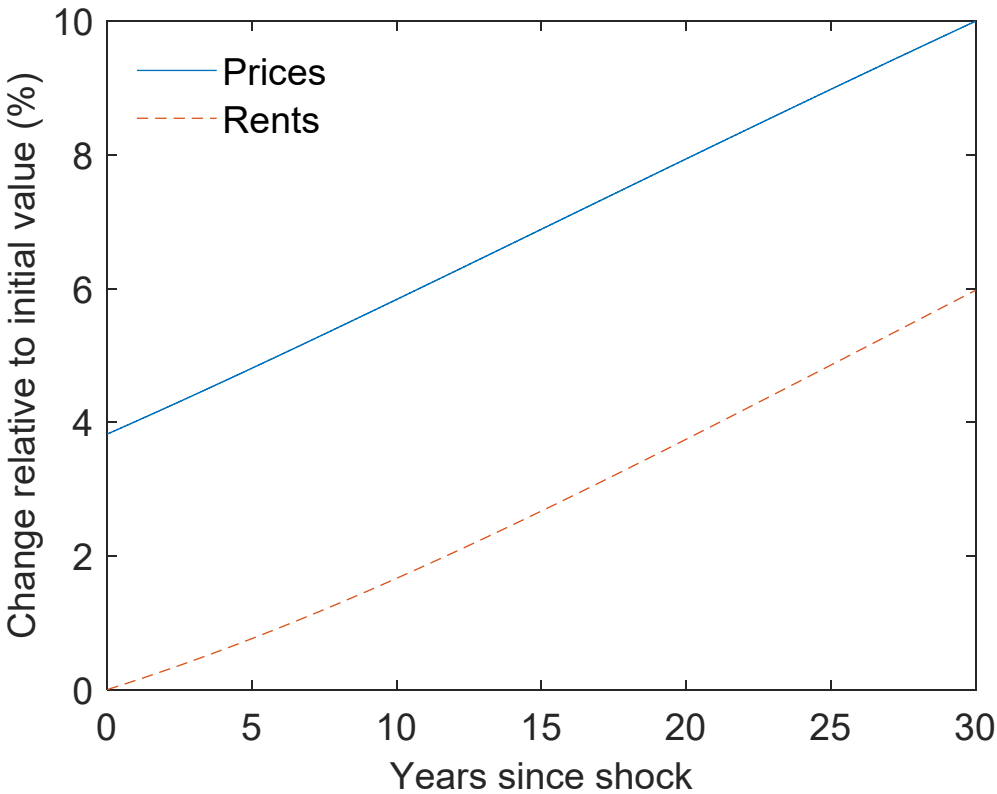
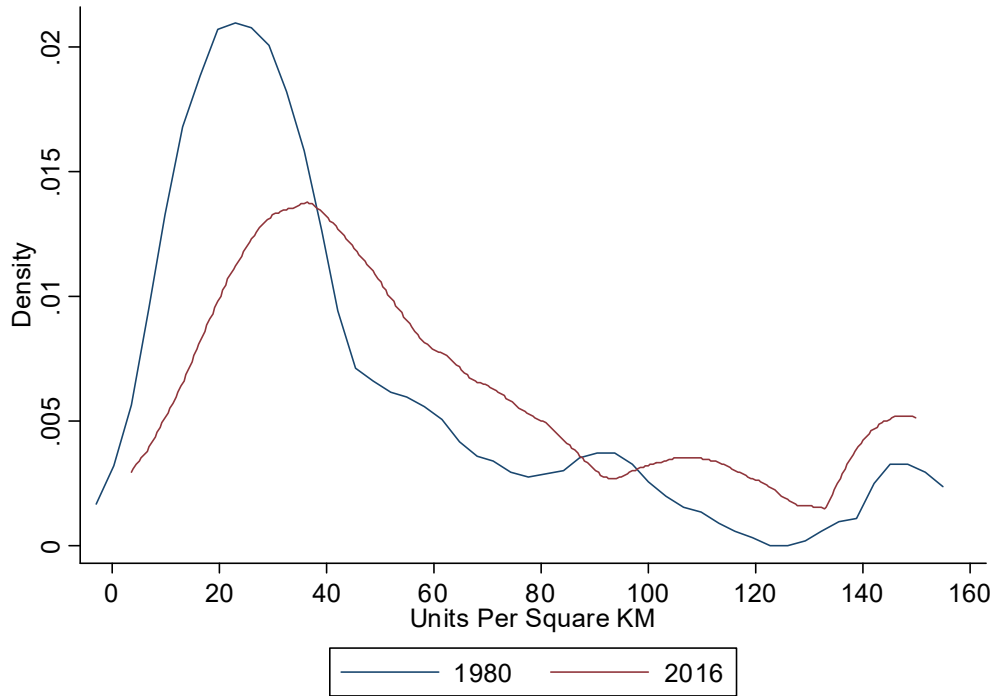
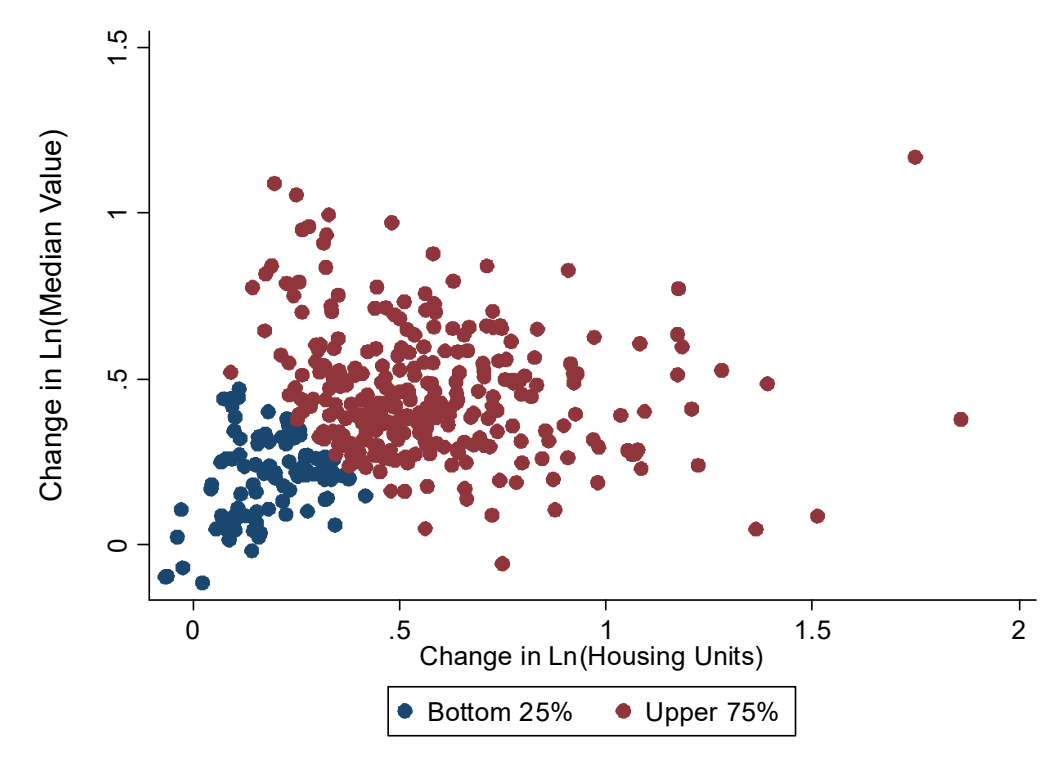


Figure 2
Distribution of Housing Unit Density
Among Central Parts of Metropolitan Areas



Note. The figure shows the distribution of housing units per square kilometer across metropolitan areas in 1980 and 2016. In each metropolitan area, density is calculated only among counties that are designated as “central” according to the 2013 OMB delineation. The sample is restricted to metropolitan areas for which not all counties are designated as central.

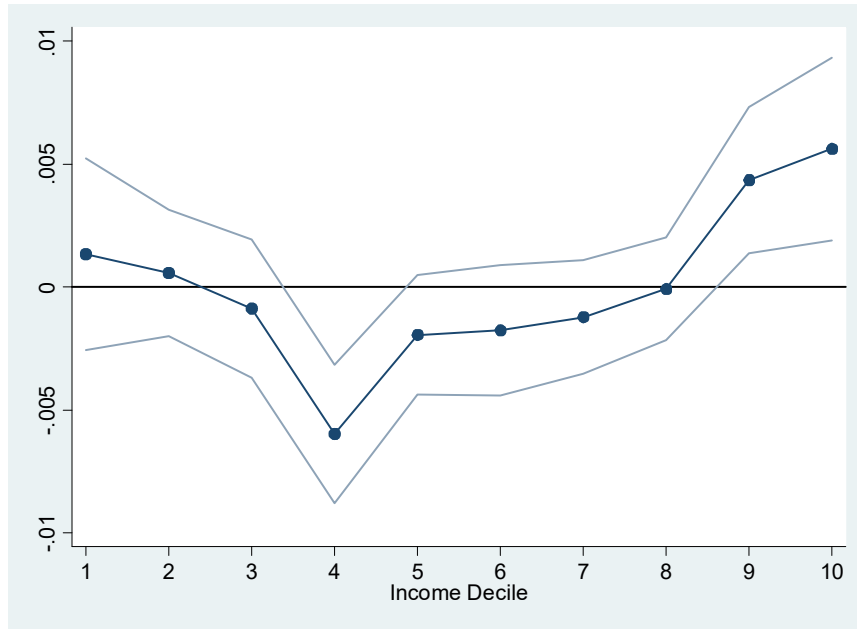
Figure 3
Identification of Low-Demand Areas Based on Growth in Housing Stock and House Value 1980-2016



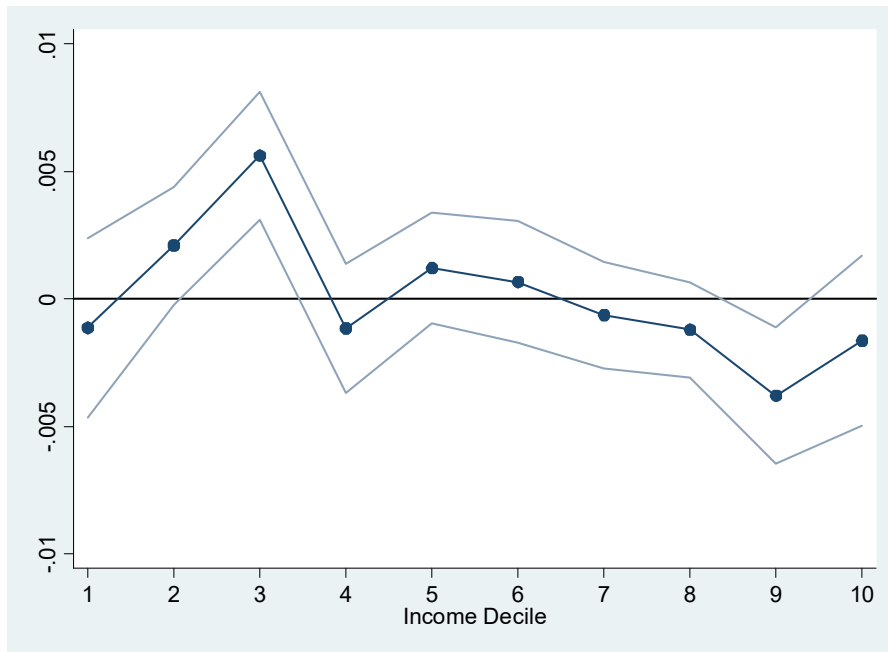
Note. Housing units include single-family and multifamily units. Median value is expressed relative to the price index for personal consumption expenditures.

Figure 4

Panel A: Effect of Regulatory Constraints on the Fraction of People in Each Income Decile

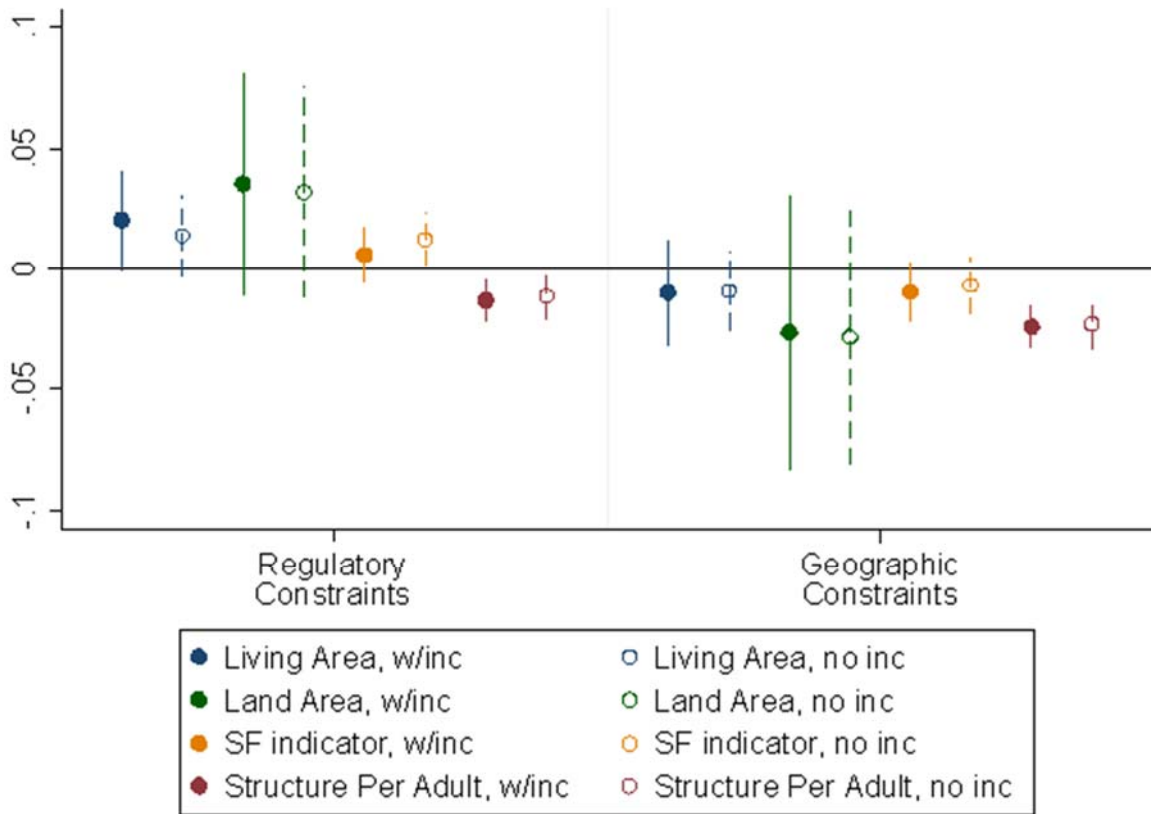


Panel B: Effect of Geographic Constraints on the Fraction of People in Each Income Decile



Note. The chart shows the estimated effects of a supply constraint on the change in the fraction of people in each decile of the national income distribution from 1980 to 2016. Regressions control for the following metro-level proxies for productivity and amenities: fraction of population with 4+ years college in 1980, fraction of employment in high-wage employment in 1990, average January temperature 1980-2010, fraction of employment in consumption-related industries in 1980, and fraction seasonal housing units in 1980.

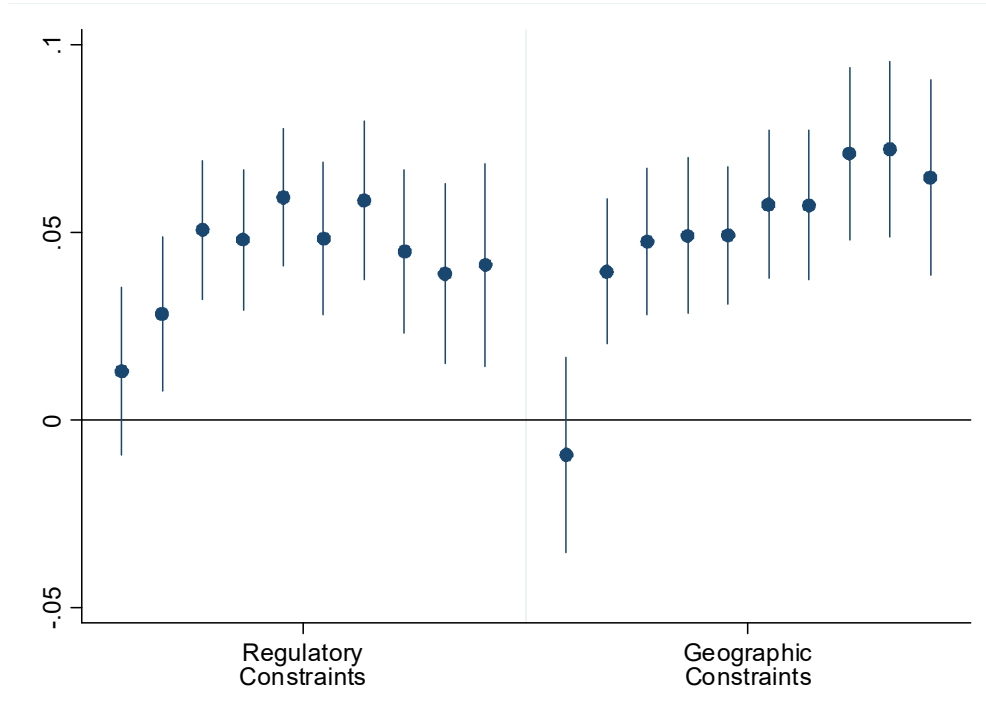
Figure 5
Comparison of Estimated Effects of Supply Constraints on Housing Consumption,
Controlling for Income Versus Not Controlling for Income



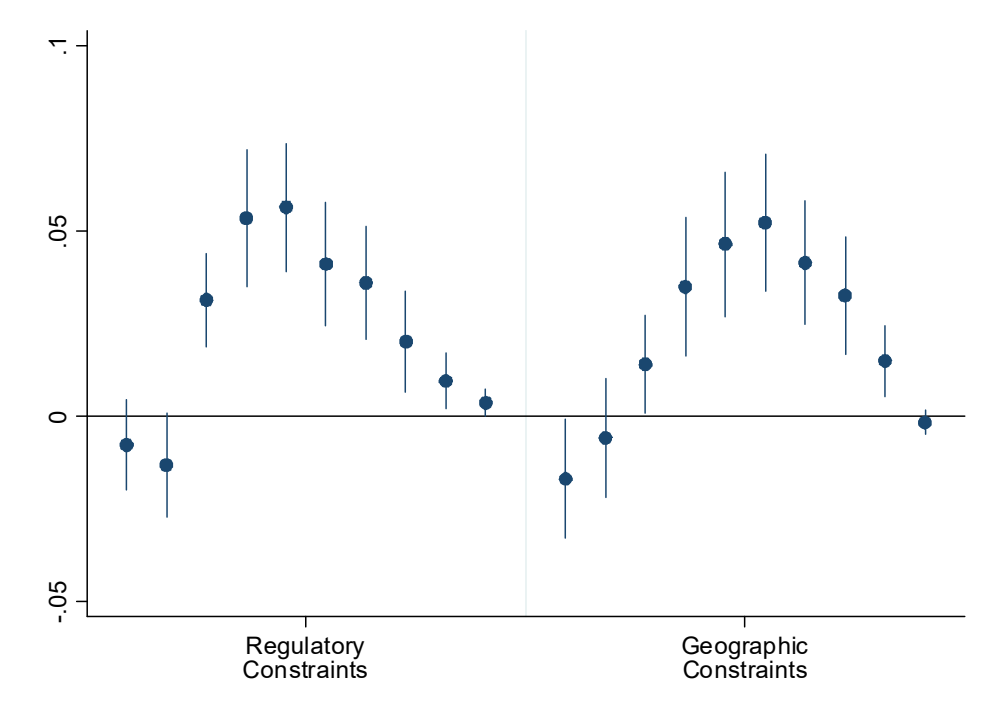
Note. The chart shows the estimated effects of a supply constraint on the dependent variables listed in the key. Regression estimates shown as solid dots are the same specifications shown in Table 4. The estimates shown as hollow dots remove the controls for income.

Figure 6

Panel A: Effect of Supply Constraints on Ln(Real Housing Expenditures) by Decile of Household Income



Panel B: Effect of Supply Constraints on Housing Expenditure Shares by Decile of Household Income



Note. The dots show coefficient estimates from regressions using the same specification as shown in Table 7, except that regressions are estimated separately for households in each decile in the national distribution of household income

Appendix Table 1
Illustration of Identification Strategy: Effects on Ln(Real House Value)

	2016	Panel: 1980 and 2016			
Regulatory Constraints	0.328 (0.070)	--	--	--	--
2012-2016 Indicator	--	0.453 (0.023)	0.453 (0.024)	0.417 (0.023)	0.487 (0.023)
Indicator interacted with:					
Regulatory constraints	--	0.183 (0.032)	0.161 (0.033)	0.127 (0.030)	0.096 (0.022)
Geographic constraints	--	--	0.074 (0.028)	0.064 (0.025)	0.119 (0.022)
Share with 4+ years college	--	--	--	0.054 (0.021)	-0.008 (0.027)
Share in high wage growth industries	--	--	--	0.037 (0.023)	0.035 (0.021)
January temperature	--	--	--	0.024 (0.026)	-0.047 (0.026)
Share employed in local consumption amenities	--	--	--	-0.100 (0.037)	-0.096 (0.027)
Share seasonal housing units	--	--	--	0.089 (0.056)	0.000 (0.042)
Exclude low-demand areas?	No	No	No	No	Yes
Number of observations	2.7 mil.	3.0 mil.	3.0 mil.	2.9 mil.	2.4 mil.

Note. Observations are owner-occupied housing units. House value is deflated with the price index for personal consumption expenditures. Standard errors in columns 2 to 5 are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. All columns control for a single-family indicator, indicators for decade built, indicators for number of rooms and indicators for number of bedrooms.

Appendix Table 2
Illustration of Identification Strategy: Effects on Housing Stock

	Ln(Stock[2016])	Ln(Housing Stock[2016]/Housing Stock[1980])					
Regulatory Constraints	0.327 (0.065)	--	--	--	--	--	--
2012-2016 Indicator	--	0.436 (0.016)	0.436 (0.016)	0.465 (0.015)	0.546 (0.017)	0.532 (0.019)	0.528 (0.021)
Indicator interacted with:							
Regulatory constraints	--	0.055 (0.016)	0.066 (0.017)	0.032 (0.015)	-0.016 (0.017)	-0.016 (0.019)	-0.035 (0.020)
Geographic constraints	--	--	-0.039 (0.017)	-0.062 (0.014)	-0.080 (0.017)	-0.082 (0.018)	-- --
Share with 4+ years college	--	--	--	0.053 (0.016)	0.011 (0.017)	0.017 (0.019)	0.019 (0.021)
Share in high wage growth industries	--	--	--	0.017 (0.016)	0.023 (0.016)	0.031 (0.018)	0.019 (0.019)
January temperature	--	--	--	0.135 (0.014)	0.107 (0.016)	0.123 (0.019)	0.109 (0.020)
Share employed in local consumption amenities	--	--	--	0.080 (0.017)	0.090 (0.017)	0.088 (0.018)	0.074 (0.019)
Share seasonal housing units	--	--	--	0.091 (0.028)	0.065 (0.029)	0.070 (0.031)	0.041 (0.033)
Exclude low-demand areas?	No	No	No	No	Yes	Yes	Yes
Exclude metros not identified in public use data files?	No	No	No	No	No	Yes	Yes
Number of observations	252	252	251	217	154	133	133

Note. Observations are metropolitan areas. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. The housing stock includes single-family and multifamily units.

Appendix Table 3
Effect of Housing Supply Constraints on House Prices and Rent
Metro Areas Without Rent Control

	Ln(Value) SF Homes	Ln(Rent) SF homes	Ln(Rent) All homes	Ln(Rent) 2-Bed Apt.
2012-2016 Indicator	0.509 (0.022)	0.504 (0.016)	0.496 (0.015)	0.465 (0.017)
Indicator interacted with:				
Regulatory constraints	0.103 (0.027)	0.047 (0.011)	0.041 (0.009)	0.051 (0.013)
Geographic constraints	0.076 (0.018)	0.015 (0.011)	0.034 (0.010)	0.034 (0.013)
Controls for Housing Characteristics	Yes	Yes	Yes	Yes
Controls for metro productivity and amenities	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes
Outcome Data	Census/ACS	Census/ACS	Census/ACS	Census/ACS
Number of Observations	1.7 million	0.29 million	0.82 million	0.23 million

Note. Standard errors are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. Controls for housing characteristics are indicators for decade built, indicators for number of rooms, and indicators for number of bedrooms. Controls for metro productivity and amenities are the following variables interacted with the “recent” indicator: fraction of population with 4+ years college in 1980, fraction of employment in high-wage employment in 1990, average January temperature 1980-2010, fraction of employment in consumption-related industries in 1980, and fraction seasonal housing units in 1980. Value and rent are expressed relative to the price index for personal consumption expenditures. Metropolitan areas with rent control are identified from http://www.landlord.com/rent_control_laws_by_state.htm.

Appendix Table 4
Effect of Housing Supply Constraints on House Prices and Rent
Household Head Moved In Within Five Years

	Ln(Value) SF Homes	Ln(Rent) SF homes	Ln(Rent) All homes	Ln(Rent) 2-Bed Apt.
2012-2016 Indicator	0.418 (0.024)	0.465 (0.015)	0.485 (0.015)	0.457 (0.018)
Indicator interacted with:				
Regulatory constraints	0.083 (0.024)	0.037 (0.012)	0.044 (0.012)	0.053 (0.014)
Geographic constraints	0.114 (0.020)	0.035 (0.011)	0.064 (0.013)	0.067 (0.015)
Controls for Housing Characteristics	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes
Outcome Data	Census/ACS	Census/ACS	Census/ACS	Census/ACS
Number of Observations	0.50 million	0.27 million	0.86 million	0.25 million

Note. Standard errors are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. Controls for housing characteristics are indicators for decade built, indicators for number of rooms, and indicators for number of bedrooms. Controls for metro productivity and amenities are the following variables interacted with the “recent” indicator: fraction of population with 4+ years college in 1980, fraction of employment in high-wage employment in 1990, average January temperature 1980-2010, fraction of employment in consumption-related industries in 1980, and fraction seasonal housing units in 1980. Value and rent are expressed relative to the price index for personal consumption expenditures.

Appendix Table 5
Effect of Housing Supply Constraints on Single-Family Indicator

	Baseline	Recently Built Homes	Home Owners	Renters	Young Owners	Young Renters
2012-2016 Indicator	0.107 (0.007)	0.181 (0.010)	0.040 (0.004)	0.093 (0.008)	0.015 (0.006)	0.068 (0.008)
Indicator interacted with:						
Regulatory constraints	0.006 (0.006)	0.003 (0.007)	0.001 (0.003)	0.001 (0.006)	-0.015 (0.004)	-0.000 (0.006)
Geographic constraints	-0.010 (0.006)	-0.027 (0.006)	-0.004 (0.003)	-0.017 (0.007)	-0.015 (0.005)	-0.026 (0.006)
Controls for Income	Yes	Yes	Yes	Yes	Yes	Yes
Controls for metro productivity and amenities	Yes	Yes	Yes	Yes	Yes	Yes
Metro Area Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Number of Observations	3.7 mil.	0.65 mil.	2.4 mil.	1.3 mil.	0.5 mil.	0.6 mil.

Note. Standard errors are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. Controls for income are indicators for decile in the national distribution of household income and these indicators interacted with a “recent” indicator. Controls for metro productivity and amenities are the following variables interacted with the “recent” indicator: fraction of population with 4+ years college in 1980, fraction of employment in high-wage employment in 1990, average January temperature 1980-2010, fraction of employment in consumption-related industries in 1980, and fraction seasonal housing units in 1980. Recently built are homes built post-1969 for the 1980 Census sample and homes built post-1999 for the 2016 ACS sample. Young households are those with a household head age 40 or less.

Appendix Table 6
Effect of Housing Supply Constraints on Neighborhood Population Growth

	1980-2010 Population Growth
Ln(Distance to Metro Center)	0.214 (0.048)
Interacted with:	
Regulatory Constraints	0.005 (0.021)
Geographic Constraints	-0.048 (0.020)
Ln(Average Commute Time)	0.175 (0.050)
Interacted with:	
Regulatory Constraints	0.036 (0.020)
Geographic Constraints	-0.057 (0.026)
Education Index	0.142 (0.051)
Interacted with:	
Regulatory Constraints	-0.040 (0.030)
Geographic Constraints	-0.116 (0.045)
Crime Index	-0.119 (0.045)
Interacted with:	
Regulatory Constraints	-0.015 (0.018)
Geographic Constraints	0.039 (0.019)
Controls for Income	Yes
Controls for metro productivity and amenities	Yes
Metro Area Fixed Effects	Yes
Outcome Data	Tract
Number of Observations	40,402

Note. Standard errors are clustered by metropolitan area. All reported right-hand-side variables are standardized to have a mean equal to zero and standard deviation equal to one. Controls for income are median household income by Census tract interacted with tract characteristics. Controls for metro productivity and amenities are the following variables interacted with the “recent” indicator: fraction of population with 4+ years college in 1980, fraction of employment in high-wage employment in 1990, average January temperature 1980-2010, fraction of employment in consumption-related industries in 1980, and fraction seasonal housing units in 1980.

A. Appendix

A.1. CES specification

In the most general form of the model, both preferences and the housing production function take the CES functional form:

$$v(c_{i,t}, h_{i,t}) = \left((1 - \alpha)c_{i,t}^\rho + \alpha h_{i,t}^\rho \right)^{\frac{1}{\rho}}$$

and

$$h(q, l) = (\gamma q^\sigma + (1 - \gamma)l^\sigma)^{\frac{1}{\sigma}},$$

where $\rho, \sigma \leq 1$. When either parameter approaches zero, the corresponding CES function limits to the Cobb-Douglas specification in the text. We first solve the model under this generality and reduce to the Cobb-Douglas case to prove the results in the text.

A.2. Developer optimization

The developer problem is

$$\min_{q,l} k^q q + (p_{j,t}^l + k^l)l + x_{j,t}$$

subject to the constraint $(\gamma q^\sigma + (1 - \gamma)l^\sigma)^{\frac{1}{\sigma}} \geq h$. Solving this gives the price of housing: $p_{j,t}(h) = x_{j,t} + m_{j,t}h$, where

$$m_{j,t} = \left((1 - \gamma)^{\frac{1}{1-\sigma}} (p_{j,t}^l + k^l)^{-\frac{\sigma}{1-\sigma}} + \gamma^{\frac{1}{1-\sigma}} (k^q)^{-\frac{\sigma}{1-\sigma}} \right)^{-\frac{1-\sigma}{\sigma}}.$$

The minimizing structure size is

$$q_{j,t}(h) = \gamma^{-\frac{1}{\sigma}} \left(1 + \left(\frac{1 - \gamma}{\gamma} \right)^{\frac{1}{1-\sigma}} \left(\frac{k^q}{p_{j,t}^l + k^l} \right)^{\frac{\sigma}{1-\sigma}} \right)^{-\frac{1}{\sigma}} h,$$

and the minimizing lot size is

$$l_{j,t}(h) = (1 - \gamma)^{-\frac{1}{\sigma}} \left(1 + \left(\frac{\gamma}{1 - \gamma} \right)^{\frac{1}{1-\sigma}} \left(\frac{p_{j,t}^l + k^l}{k^q} \right)^{\frac{\sigma}{1-\sigma}} \right)^{-\frac{1}{\sigma}} h.$$

When $x_{j,t} = p_{j,t}^l = 0$, the budget share of structure is

$$\tilde{\gamma} = \frac{(k^l)^{\frac{\sigma}{1-\sigma}} \gamma^{\frac{1}{1-\sigma}}}{(k^l)^{\frac{\sigma}{1-\sigma}} \gamma^{\frac{1}{1-\sigma}} + (k^q)^{\frac{\sigma}{1-\sigma}} (1-\gamma)^{\frac{1}{1-\sigma}}}.$$

A.3. Dynasty optimization

Here we solve the dynasty problem given beliefs about future prices. We apply this solution to proofs that follow. Let λ_d denote the multiplier on the dynasty budget constraint for dynasty d . The first-order conditions for $c_{i,t}$ and $h_{i,t}$ respectively are

$$\lambda_d = \frac{(1-\alpha)c_{i,t}^{\rho-1}}{(1-\alpha)c_{i,t}^\rho + \alpha h_{i,t}^\rho}$$

and

$$\lambda_d m_{j,t} = \frac{\alpha h_{i,t}^{\rho-1}}{(1-\alpha)c_{i,t}^\rho + \alpha h_{i,t}^\rho}.$$

The solution depends only on d, j , and t , so consumption levels are identical across households in the same city and dynasty at the same time. This solution is

$$c_{d,j,t} = \frac{\lambda_d^{-1} (1-\alpha)^{\frac{1}{1-\rho}}}{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{j,t}^{\frac{\rho}{\rho-1}}}$$

and

$$h_{d,j,t} = \frac{\lambda_d^{-1} \alpha^{\frac{1}{1-\rho}} m_{j,t}^{\frac{1}{\rho-1}}}{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{j,t}^{\frac{\rho}{\rho-1}}}.$$

The optimized flow utility is

$$v_{d,j,t} = \lambda_d^{-1} \left((1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{j,t}^{\frac{\rho}{\rho-1}} \right)^{\frac{1-\rho}{\rho}}.$$

Expenditure for households in d and j at t is $c_{d,j,t} + x_{j,t} + m_{j,t} h_{d,j,t} = x_{j,t} + \lambda_d^{-1}$. The dynastic budget constraint therefore reduces to

$$\frac{y_d}{r-g} = \frac{\lambda_d^{-1}}{r-g} + \int_0^\infty e^{-(r-g)t} s_{d,R,t} x_{R,t} dt,$$

where $s_{d,j,t}$ equals the share of households in dynasty d in city j at t . Therefore

$$\lambda_d = \left(y_d - \int_0^\infty (r-g)e^{-(r-g)t} s_{d,R,t} x_{R,t} dt \right)^{-1}.$$

Substitution into dynastic utility transforms it to

$$(r-g)^{-1} \log \left(y_d - \int_0^\infty (r-g)e^{-(r-g)t} s_{d,R,t} x_{R,t} dt \right) + \int_0^\infty e^{-(r-g)t} \sum_{j \in \{F,R\}} s_{d,j,t} \left(\log a_{d,j} + \frac{1-\rho}{\rho} \log \left((1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{j,t}^{\frac{\rho}{\rho-1}} \right) \right) dt$$

times the number of households in the dynasty at time zero, where $a_{d,j}$ is the common taste for city j across households in dynasty d . Because $s_{d,F,t} = 1 - s_{d,R,t}$, the marginal gain from increasing $s_{d,R,t}$ is

$$\log \left(\frac{a_{d,R}}{a_{d,F}} \right) - \frac{1-\rho}{\rho} \log \left(\frac{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{F,t}^{\frac{\rho}{\rho-1}}}{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{R,t}^{\frac{\rho}{\rho-1}}} \right) - \frac{x_{R,t}}{y_d - (r-g) \int_0^\infty e^{-(r-g)t'} s_{d,R,t'} x_{R,t'} dt'}$$

times $e^{-(r-g)t}$. When this gain is positive, $s_{d,R,t} = 1$, when this gain is negative, $s_{d,R,t} = 0$, and when this gain equals zero, $s_{d,R,t}$ can take any value between zero and 1.

A.4. Initial equilibrium

At time zero, households believe that city R will remain unregulated. Therefore, they believe that $x_{R,t} = p_{R,t}^l = 0$ for all $t \geq 0$. As a result, $p_{R,0}(h) = m_0 h$ and $p_{R,0}^{own} = r^{-1} m_0 h$, where

$$m_0 = \left((1-\gamma)^{\frac{1}{1-\sigma}} (k^l)^{-\frac{\sigma}{1-\sigma}} + \gamma^{\frac{1}{1-\sigma}} (k^q)^{-\frac{\sigma}{1-\sigma}} \right)^{-\frac{1-\sigma}{\sigma}}.$$

Furthermore, $s_{d,R,t} = 1$ when $a_{d,R} > a_{d,F}$ and $s_{d,R,t} = 0$ when $a_{d,R} < a_{d,F}$. The initial population in R equals the measure of households for whom $a_{i,R} > a_{i,F}$, which reduces to $\epsilon_{i,R} - \epsilon_{i,F} > \beta(\log a_R - \log a_F)$. Given standard results about extreme value distributions, the population is

$$N_{R,0} = a_R^\beta (a_F^\beta + a_R^\beta)^{-1} N_0.$$

From Section A.2, the lot size of a house in R is $l_{R,0} h = (1 - \tilde{\gamma})(k^l)^{-1} m_0 h$, and from Section A.3, housing consumption for household i is $h_{i,0} = m_0^{-1} \tilde{\alpha} y_i$, where

$$\tilde{\alpha} = \frac{\alpha^{\frac{1}{1-\rho}} m_0^{\frac{\rho}{\rho-1}}}{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_0^{\frac{\rho}{\rho-1}}}.$$

The lot size for household i in city R is $\tilde{\alpha}(1 - \tilde{\gamma})(k^l)^{-1}y_i$. The total land area of R at time zero is

$$L_{R,0} = \tilde{\alpha}(1 - \tilde{\gamma})(k^l)^{-1}a_R^\beta (a_F^\beta + a_R^\beta)^{-1} N_0 \bar{y},$$

where $\bar{y} = \int_0^\infty yf(y)dy$.

A.5. Proof of Proposition 1

By the $s_{d,R,t}$ condition from Section A.3, a household with $a_{i,R} < a_{i,F}$ always lives in F . Otherwise, given $s_{d,R,t}$, there exists a unique x_i^* such that the household lives in R if $x_{R,t} < x_i^*$ and lives in F if $x_{R,t} > x_i^*$. As a result, the household lives in R when

$$\log\left(\frac{a_{i,R}}{a_{i,F}}\right) \geq \frac{x_{R,t}}{y_i - (r - g) \int_{(t|x \leq x_i^*)} e^{-(r-g)t} x_{R,t} dt}.$$

This inequality is an equality when $x_{R,t} = x_i^*$, which proves the proposition. This proposition holds under the CES specification as well.

A.6. Proof of Proposition 2

As $x_{R,t}$ increases, $\bar{x}(x_{R,t})$ weakly increases because the domain of integration in the definition of \bar{x} weakly expands. The right side of the equation that we claim determines $x_{R,t}$ therefore strictly decreases in $x_{R,t}$. Because the left side of this equation strictly decreases in t , $x_{R,t}$ must strictly decrease in t , as claimed.

The equations for rent and price changes are immediate from substituting the equation for $p_{R,t}(h)$ for $t > 0$ from section 2.1 into the equation for $p_{R,0}(h)$ from appendix A.4 (taking the limit as $\sigma \rightarrow 0$). Because $r \int_t^\infty e^{-r(t'-t)} x_{R,t'} dt'$ averages $x_{R,t'}$ over the interval $[t, \infty)$, this average strictly exceeds $x_{R,t}$ because $x_{R,t'}$ increases in t' . As a result, prices rise more than rents.

A.7. Proof of Proposition 3

Using the formulas from appendix A.3 in the $\rho \rightarrow 0$ limit, we have

$$h_i^* = \alpha \gamma^\gamma (1 - \gamma)^{1-\gamma} (k^l)^{\gamma-1} (k^q)^{-\gamma} (y_i - \bar{x}(x_i^*)).$$

Due to Cobb-Douglas production, the share of the value going to structure is γ and the share going to lot is $1 - \gamma$. We substitute h_i^* into the expression for housing spot price from section 2.1 and multiply by $\gamma(k^q)^{-1}$ and $(1 - \gamma)(k^l)^{-1}$ to obtain q_i^* and l_i^* , respectively.

We now show that

$$E(y_i - \bar{x}(x_i^*) \mid y_i) < E(y_{i'} - \bar{x}(x_{i'}^*) \mid y_{i'}).$$

If $y_i < y_{i'}$ and both households are in city R at time t . Doing so proves the final statement in the proposition. By Proposition 1, households of income y_i reside in R at t only if

$$\log\left(\frac{a_{R,i}}{a_{F,i}}\right) \geq \frac{x_{R,t}}{y_i - \bar{x}(x_{R,t})}.$$

Call this threshold ϕ_i . We have $\phi_i > \phi_{i'}$. If $a_{R,i}/a_{F,i} = a_{R,i'}/a_{F,i'}$, then $x_i^* < x_{i'}^*$ which means that $y_i - \bar{x}(x_i^*) < y_{i'} - \bar{x}(x_{i'}^*)$ by Proposition 1. The distribution of $\log(a_{R,i}/a_{F,i})$ and $\log(a_{R,i'}/a_{F,i'})$ is the same conditional on exceeding ϕ_i . Therefore, for such households, our claim holds. Furthermore, because $\bar{x}(x_i^*)$ rises in $\log(a_{R,i'}/a_{F,i'})$, $y_{i'} - \bar{x}(x_{i'}^*)$ is larger when $\log(a_{R,i'}/a_{F,i'}) \leq \phi_i$ than when $\log(a_{R,i'}/a_{F,i'}) > \phi_i$. Therefore, the inequality we desire holds for the entire distribution.

A.8. Proof of Proposition 4

This proposition follows immediately from the $s_{d,R,t}$ condition from section A.3.

A.9. Proof of Proposition 5

Using the formulas from appendix A.3 in the $\rho \rightarrow 0$ and $\sigma \rightarrow 0$ limit, we have

$$h_i^* = \alpha\gamma^\gamma(1-\gamma)^{1-\gamma}(k^l + p_{R,t}^l)^{\gamma-1}(k^q)^{-\gamma}y_i.$$

Due to Cobb-Douglas production, the share of the value going to structure is γ and the share going to lot is $1 - \gamma$. We substitute h_i^* into the expression for housing spot price from section 2.1 and the multiply by $\gamma(k^q)^{-1}$ and $(1-\gamma)(k^l)^{-1}$ to obtain q_i^* and l_i^* , respectively.

A.10. Proof of Proposition 6

The equations for rent and price changes are immediate from substituting the equation for $p_{R,t}(h)$ for $t > 0$ from section 2.1 into the equation for $p_{R,0}(h)$ from appendix A.4. Because the price effect averages the current and future rent effects, which strictly increase over time, the price effect exceeds the rent effect.

A.11. Quantitative model solution

In the case of permit delays, we solve for $x_{R,t}$ using differential equations as follows. We define $\bar{x}_t = (r-g) \int_0^t e^{-(r-g)t'} x_{R,t'} dt'$, which equals $\bar{x}(x_{R,t})$ because $x_{R,t}$ strictly increases. Differentiation yields

$$\dot{\bar{x}}_t = (r-g)e^{-(r-g)t} x_{R,t}.$$

Because Proposition 1 holds in the general CES case, the equation from section 2.2 determining $x_{R,t}$ holds in the CES model as well. Differentiating this equation gives

$$(g - g^n)e^{-(g-g^n)t} = \int_{\bar{x}_t}^{\infty} \frac{\beta(a_F^\beta + a_R^\beta)a_F^\beta \exp\left(\frac{\beta x_{R,t}}{y - \bar{x}_t}\right) \left((y - \bar{x}_t)\dot{x}_{R,t} + x_{R,t}\dot{\bar{x}}_t\right)}{\left(a_F^\beta \exp\left(\frac{\beta x_{R,t}}{y - \bar{x}_t}\right) + a_R^\beta\right)^2 (y - \bar{x}_t)^2} f(y) dy.$$

We then substitute the equation for $\dot{\bar{x}}_t$ and solve for $\dot{x}_{R,t}$ to obtain

$$\dot{x}_{R,t} = \left(\frac{(g - g^n)e^{-(g-g^n)t}}{\beta(a_F^\beta + a_R^\beta)a_F^\beta} - x_{R,t}(r - g)e^{-(r-g)t} \right) \left(\int_{\bar{x}_t}^{\infty} \frac{\exp\left(\frac{\beta x_{R,t}}{y - \bar{x}_t}\right) f(y) dy}{\left(a_F^\beta \exp\left(\frac{\beta x_{R,t}}{y - \bar{x}_t}\right) + a_R^\beta\right)^2 (y - \bar{x}_t)} \right)^{-1}.$$

We now have two differential equations in the two unknowns $x_{R,t}$ and \bar{x}_t . The initial conditions are $x_{R,0} = 0$ and $\bar{x}_0 = 0$.

In the case of geographic constraints, we calculate $m_{R,t}$ numerically. In the Cobb-Douglas case, we use the explicit formula for $p_{R,t}^l$ appearing in section 2.3. In the CES case, we make a series of substitutions to derive differential equations pinning down this price over time from the market-clearing condition for land. Define

$$u_t = \left(\frac{m_{R,t}}{m_0} \right)^{\frac{\rho}{\rho-1}}$$

and

$$v_t = \frac{(1 - \alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_0^{\frac{\rho}{\rho-1}}}{(1 - \alpha)^{\frac{1}{1-\rho}} + u_t \alpha^{\frac{1}{1-\rho}} m_0^{\frac{\rho}{\rho-1}}}.$$

From Section A.3, the measure of households choosing R is the measure of those for whom

$$\log\left(\frac{a_{d,R}}{a_{d,F}}\right) \geq \frac{1 - \rho}{\rho} \log\left(\frac{(1 - \alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{F,t}^{\frac{\rho}{\rho-1}}}{(1 - \alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}} m_{R,t}^{\frac{\rho}{\rho-1}}}\right),$$

which equals

$$s_{R,t} = \frac{a_R^\beta}{a_R^\beta + a_F^\beta v_t^{\frac{\beta(1-\rho)}{\rho}}}.$$

We also define

$$w_t = \left(\frac{(1-\gamma)^{\frac{1}{1-\sigma}}(k^q)^{\frac{\sigma}{1-\sigma}} + \gamma^{\frac{1}{1-\sigma}}(k^l)^{\frac{\sigma}{1-\sigma}}}{(1-\gamma)^{\frac{1}{1-\sigma}}(k^q)^{\frac{\sigma}{1-\sigma}} + \gamma^{\frac{1}{1-\sigma}}(p_{R,t}^l + k^l)^{\frac{\sigma}{1-\sigma}}} \right)^{\frac{1}{\sigma}}$$

and

$$z_t = \left(\frac{p_{R,t}^l + k^l}{k^l} \right)^{\frac{\sigma}{1-\sigma}}.$$

Using these substitutions, we write the lot size of household i in city R as

$$l_{i,R} = \left(\frac{1-\tilde{\gamma}}{1-\gamma} \right)^{\frac{1}{\sigma}} w_t u_t^{\frac{1}{\rho}} \tilde{\alpha} v_t y_i.$$

Market-clearing then simplifies to

$$e^{-(g-g^l)t} = w_t u_t^{\frac{1}{\rho}} v_t \frac{a_R^\beta + a_F^\beta}{a_R^\beta + a_F^\beta v_t^{\frac{\beta(1-\rho)}{\rho}}}.$$

This equation holds at all $t \geq 0$ and pins down $m_{R,t}$. It holds at $t = 0$ because $u_0 = v_0 = w_0 =$

1. Log differentiation yields

$$-(g-g^l) = \frac{\dot{w}_t}{w_t} + \frac{1}{\rho} \frac{\dot{u}_t}{u_t} + \frac{\dot{v}_t}{v_t} \left(1 - \frac{\frac{\beta(1-\rho)}{\rho} a_F^\beta v_t^{\frac{\beta(1-\rho)}{\rho}}}{a_R^\beta + a_F^\beta v_t^{\frac{\beta(1-\rho)}{\rho}}} \right).$$

We have

$$\frac{\dot{u}_t}{u_t} = \frac{\rho}{\rho-1} \frac{\dot{m}_{R,t}}{m_{R,t}},$$

$$\frac{\dot{v}_t}{v_t} = -\dot{u}_t v_t \tilde{\alpha},$$

$$\frac{\dot{m}_{R,t}}{m_{R,t}} = \frac{\dot{p}_{R,t}^l + k^l}{p_{R,t}^l + k^l} (1-\tilde{\gamma}) w_t^\sigma,$$

$$\frac{\dot{w}_t}{w_t} = -\frac{1}{1-\sigma} \frac{\dot{p}_{R,t}^l + k^l}{p_{R,t}^l + k^l} z_t \tilde{\gamma} w_t,$$

and

$$\frac{\dot{z}_t}{z_t} = -\frac{\sigma}{1-\sigma} \frac{1}{1-\tilde{\gamma}} w_t^{-\sigma} \frac{1-\rho}{\rho} \frac{\dot{u}_t}{u_t}.$$

That gives us six differential equations in six unknowns: $u_t, v_t, w_t, z_t, \log m_{R,t}/m_0$, and $\log(1 + p_{R,t}^l/k^l)$. Solving these gives us $m_{R,t}$ at all times. From that we solve all other variables.